UPR-1068-T, hep-th/0403061

Supersymmetric Pati-Salam Models from Intersecting D6-branes: A Road to the Standard Model

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(Dated: February 7, 2008)

Abstract

We provide a systematic construction of three-family $N = 1$ supersymmetric Pati-Salam models from Type IIA orientifolds on $\mathbf{T^6}/(\mathbf{Z_2} \times \mathbf{Z_2})$ with intersecting D6-branes. All the gauge symmetry factors $SU(4)_C \times SU(2)_L \times SU(2)_R$ arise from the stacks of D6-branes with $U(n)$ gauge symmetries, while the "hidden sector" is specified by $USp(n)$ branes, parallel with the orientifold planes or their $\mathbf{Z_2}$ images. The Pati-Salam gauge symmetry can be broken down to the $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ via D6-brane splittings, and further down to the Standard Model via D- and F-flatness preserving Higgs mechanism from massless open string states in a $N = 2$ subsector. The models also possess at least two confining hidden gauge sectors, where gaugino condensation can in turn trigger supersymmetry breaking and (some) moduli stabilization. The systematic search yields 11 inequivalent models: 8 models with less than 9 Standard model Higgs doublet-pairs and 1 model with only 2 Standard Model Higgs doublet-pairs, 2 models possess at the string scale the gauge coupling unification of $SU(2)_L$ and $SU(2)_R$, and all the models possess additional exotic matters. We also make preliminary comments on phenomenological implications of these models.

I. INTRODUCTION

Prior to the second string revolution the efforts in string phenomenology focused on constructions of four-dimensional solutions in the weakly coupled heterotic string theory: the goal was to construct $N = 1$ supersymmetric models with features of the Standard Model (SM). On the other hand, the M-theory unification possesses in addition to its perturbative heterotic string theory corner, also other corners such as perturbative Type I, Type IIA and Type IIB superstring theory, which should provide new potentially phenomenologically interesting four-dimensional string solutions, related to the heterotic ones via a web of string dualities. In particular, the advent of D-branes [1], as boundaries of open strings, plays an important role in constructions of phenomenologically interesting models in Type I, Type IIA and Type IIB string theories. Conformal field theory techniques in the open string sectors, which end on D-branes, allow for exact constructions of consistent 4-dimensional supersymmetric $N = 1$ chiral models with non-Abelian gauge symmetry on Type II orientifolds. Within this framework chiral matters can appear (i) due to D-branes located at orbifold singularities with chiral fermions appearing on the worldvolume of such D-branes [2−8] and/or (ii) at the intersections of D-branes in the internal space [9] (These latter models also have a T-dual description in terms of magnetized D-branes [10, 11].).

Within the models with intersecting D6-brane on Type IIA orientifolds [12−14], a large number of non-supersymmetric three-family Standard-like models and grand unified models were constructed [12−25]. These models satisfy the Ramond-Ramond (RR) tadpole cancellation conditions, however, since the models are non-supersymmetric, there are uncancelled Neveu-Schwarz-Neveu-Schwarz (NS-NS) tadpoles. In addition, the string scale is close to the Planck scale because the intersecting D6-branes typically have no common transverse direction in the internal space. Therefore, these models typically suffer from the large Planck scale corrections at the loop level, *i.e.*, there exists the gauge hierarchy problem.

On the other hand, the supersymmetric models [26, 27] with quasi-realistic features of the supersymmetric Standard-like models have been constructed in Type IIA theory on $T^6/(\mathbf{Z_2} \times \mathbf{Z_2})$ orientifold with intersecting D6-branes. Subsequently, a larger set of supersymmetric Standard-like models and a Pati-Salam model [28], as well as a systematic construction of supersymmetric $SU(5)$ Grand Unified models [29] have been constructed. Their phenomenological consequences, such as renormalization group running for the gauge couplings, supersymmetry breaking via gaugino condensations, moduli stabilization, and the complete Yukawa couplings that include classical and quantum contributions, have been studied [30, 31, 32, 33]. Furthermore, the supersymmetric Pati-Salam models based on Z_4 and $Z_4 \times Z_2$ orientifolds with intersecting D6-branes were also constructed [34, 35]. In these models, the left-right gauge symmetry was obtained via brane recombinations, so the final

models do not have an explicit toroidal orientifold construction, where the conformal field theory can be applied for the calculation of the full spectrum and couplings.

We shall concentrate on constructions of supersymmetric three-family Pati-Salam models based on $T^6/(\mathbf{Z_2} \times \mathbf{Z_2})$ orientifold. Although previous constructions provided a number of supersymmetric three family examples with Standard-like gauge group (or Grand Unified group), these models have a number of phenomenological problems. Previous models had at least 8 pairs of the SM Higgs doublets and a number of exotic particles, some of them fractionally charged. In addition, in the previous supersymmetric SM constructions [26, 27, 28], there are at least two extra anomaly free $U(1)$ gauge symmetries. They could be in principle spontaneously broken via Higgs mechanism by the scalar components of the chiral superfields with the quantum numbers of the right-handed neutrinos, however they break D-flatness conditions, and thus supersymmetry, so the scale of symmetry breaking should be near the electroweak scale. On the other hand, there were typically no candidates, preserving D-flatness and F-flatness conditions, which could break these gauge symmetries at an intermediate scale. In addition, there is no gauge coupling unification. (The gauge coupling unification can be realized in the quasi-supersymmetric $U(n)^3$ models [39]. But the filler branes, which are on top of orientifold planes in these models, are anti-branes and break the supersymmetry at the string scale.) Furthermore, there exist three multiplets in the adjoint representation for each $U(n)$ group, which is a generic property for the supersymmetric and non-supersymmetric toroidal orientifold constructions because the typical three cycles wrapped by D6-branes are not rigid. One possible solution is that we consider the Calabi-Yau compactifications with rigid supersymmetric cycles, but, the calculational techniques of conformal field theory may not be applicable there. (For studies of $N = 1$ supersymmetric solutions of D-branes on Calabi-Yau manifolds see [36, 37, 38] and references therein.)

Well-motivated by the Standard Model constructions, we shall study systematically the three family $N = 1$ supersymmetric Pati-Salam models from Type IIA orientifolds on $\mathbf{T^6}/(\mathbf{Z_2} \times \mathbf{Z_2})$ with intersecting D6-branes where all the gauge symmetries come from $U(n)$ branes. On the one hand, Pati-Salam model provides a natural origin of $U(1)_{B-L}$ and $U(1)_{I_{3R}}$ both of which are generically required due to the quantum numbers of the SM fermions and the hypercharge interaction in the SM building from intersecting D6-brane scenarios. On the other hand, it also provides one road to the SM without any additional anomaly-free $U(1)$'s near the electroweak scale, which was a generic feature of the previous supersymmetric SM constructions [28].

The paper is organized in the following way. In Section II we briefly review the rules for supersymmetric model building with intersecting D6-branes on Type IIA orientifolds, the conditions for the tadpole cancellations and conditions on D6-brane configurations for $N = 1$ supersymmetry in four-dimension. We specifically focus on Type IIA theory on

$T^6/(\mathbf{Z_2} \times \mathbf{Z_2})$ orientifold.

In Section III, we discuss in detail the T-duality symmetry and its variations within the supersymmetric intersecting D6-brane model building. The first set of symmetries are general and can be applied to any concrete particle physics model building (type I), and the second set is special and only valid for the specific Pati-Salam model building (type II). We also find that any two models T-dual to each other have the same gauge couplings at string scale.

In Section IV, we study the phenomenological constraints in the construction of the supersymmetric Pati-Salam models. In particular, we highlight why the models where the Pati-Salam gauge symmetry comes from $U(n)$ branes are phenomenologically interesting. We show that the Pati-Salam gauge symmetry can be broken down to the $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ via D6-brane splittings, and further down to the Standard Model gauge symmetry via Higgs mechanism, *i.e.*, brane recombination in geometric interpretation [22], where the Higgs fields come from the massless open string states in a $N = 2$ subsector. In order to stabilize the moduli and provide a way to break supersymmetry, we require that at least two hidden sector gauge factors are confining, thus allowing for the "race-track" gaugino condensation mechanism.

In Section V, by employing the T-duality and its variations, we systematic search for the inequivalent models, and discuss the possible classes of solutions in detail. As a result, we obtain total of 11 inequivalent models, all of them arising from the orbifold with only one of the three two-tori tilted. Compared to the previous SM constructions [26, 27, 28], eight of our models have fewer pairs of the SM Higgs doublets (≤ 8). Interestingly, the gauge coupling unification for $SU(2)_L$ and $SU(2)_R$ can be achieved at string scale in two models. As explicit examples, we present the chiral spectra in the open string sector for the models I-NZ-1a, I-Z-6 and I-Z-10.

In Section VI, we briefly comment on the other potentially interesting setups. And the discussions and conclusions are given in Section VII. In Appendix, we present the D6-brane configurations and intersection numbers for supersymmetric Pati-Salam models.

II. CONDITIONS FOR SUPERSYMMETRIC MODELS FROM $T^6/(Z_2 \times Z_2)$ ORIENTIFOLDS WITH INTERSECTING D6-BRANES

The rules to construct supersymmetric models from Type IIA orientifolds on $T^6/(Z_2 \times Z_2)$ with D6-branes intersecting at generic angles, and to obtain the spectrum of massless open string states have been discussed in [27]. Following the convention in Ref. [29], we briefly review the essential points in the construction of such models.

The starting point is Type IIA string theory compactified on a $T^6/(Z_2\times Z_2)$ orientifold. We consider T^6 to be a six-torus factorized as $T^6=T^2\times T^2\times T^2$ whose complex coordinates are z_i, $i=1,\ 2,\ 3$ for the i-th two-torus, respectively. The θ and ω generators for the orbifold group $Z_2\times Z_2$, which are associated with their twist vectors $(1/2,-1/2,0)$ and $(0,1/2,-1/2)$ respectively, act on the complex coordinates of T^6 as

$$
\begin{aligned}
\theta:\ &(z_1,z_2,z_3)\to(-z_1,-z_2,z_3)\ ,\\
\omega:\ &(z_1,z_2,z_3)\to(z_1,-z_2,-z_3)\ .
\end{aligned}
\tag{1}
$$

The orientifold projection is implemented by gauging the symmetry ΩR, where Ω is world-sheet parity, and R acts as

$$
R:(z_1,z_2,z_3)\to(\overline{z}_1,\overline{z}_2,\overline{z}_3)\ .
\tag{2}
$$

So, there are four kinds of orientifold 6-planes (O6-planes) for the actions of ΩR, $\Omega R\theta$, $\Omega R\omega$, and $\Omega R\theta\omega$, respectively. To cancel the RR charges of O6-planes, we introduce stacks of N_a D6-branes, which wrap on the factorized three-cycles. Meanwhile, we have two kinds of complex structures consistent with orientifold projection for a two-torus – rectangular and tilted [13, 27, 29]. If we denote the homology classes of the three cycles wrapped by the D6-brane stacks as $n_a^i[a_i]+m_a^i[b_i]$ and $n_a^i[a_i']+m_a^i[b_i]$ with $[a_i']=[a_i]+\frac{1}{2}[b_i]$ for the rectangular and tilted tori respectively, we can label a generic one cycle by (n_a^i,l_a^i) in either case, where in terms of the wrapping numbers $l_a^i\equiv m_a^i$ for a rectangular two-torus and $l_a^i\equiv 2\tilde{m}_a^i=2m_a^i+n_a^i$ for a tilted two-torus. Note that for a tilted two-torus, $l_a^i-n_a^i$ must be even. For a stack of N_a D6-branes along the cycle (n_a^i,l_a^i), we also need to include their ΩR images $N_{a'}$ with wrapping numbers $(n_a^i,-l_a^i)$. For D6-branes on top of O6-planes, we count the D6-branes and their images independently. So, the homology three-cycles for stack a of N_a D6-branes and its orientifold image a' take the form

$$
[\Pi_a]=\prod_{i=1}^{3}\left(n_a^i[a_i]+2^{-\beta_i}l_a^i[b_i]\right),\quad [\Pi_{a'}]=\prod_{i=1}^{3}\left(n_a^i[a_i]-2^{-\beta_i}l_a^i[b_i]\right)\ ,
\tag{3}
$$

where $\beta_i=0$ if the i-th two-torus is rectangular and $\beta_i=1$ if it is tilted. And the homology three-cycles wrapped by the four O6-planes are

$$
\Omega R:[\Pi_{\Omega R}]=2^3[a_1]\times[a_2]\times[a_3]\ ,
\tag{4}
$$

$$
\Omega R\omega:[\Pi_{\Omega R\omega}]=-2^{3-\beta_2-\beta_3}[a_1]\times[b_2]\times[b_3]\ ,
\tag{5}
$$

$$
\Omega R\theta\omega:[\Pi_{\Omega R\theta\omega}]=-2^{3-\beta_1-\beta_3}[b_1]\times[a_2]\times[b_3]\ ,
\tag{6}
$$

$$
\Omega R\theta:[\Pi_{\Omega R}]=-2^{3-\beta_1-\beta_2}[b_1]\times[b_2]\times[a_3]\ .
\tag{7}
$$

TABLE I: General spectrum on intersecting D6-branes at generic angles which is valid for both rectangular and tilted two-tori. The representations in the table refer to $U(N_a/2)$, the resulting gauge symmetry [27] due to $Z_2 \times Z_2$ orbifold projection. For supersymmetric constructions, scalars combine with fermions to form chiral supermultiplets. In our convention, positive intersection numbers implie left-hand chiral supermultiplets.

Sector	**Representation**
aa	$U(N_a/2)$ vector multiplet 3 adjoint chiral multiplets
$ab+ba$	I_{ab} $(\square_a, \overline{\square}_b)$ fermions
$ab'+b'a$	$I_{ab'}$ $(\square_a, \square_b)$ fermions
$aa'+a'a$	$\frac{1}{2}(I_{aa'} - \frac{1}{2}I_{a,O6})$ $\square\square$ fermions $\frac{1}{2}(I_{aa'} + \frac{1}{2}I_{a,O6})$ $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ fermions

Therefore, the intersection numbers are

$$I_{ab} = [\Pi_a][\Pi_b] = 2^{-k}\prod_{i=1}^{3}(n_a^i l_b^i - n_b^i l_a^i) \ , \tag{8}$$

$$I_{ab'} = [\Pi_a]\,[\Pi_{b'}] = -2^{-k}\prod_{i=1}^{3}(n_a^i l_b^i + n_b^i l_a^i) \ , \tag{9}$$

$$I_{aa'} = [\Pi_a]\,[\Pi_{a'}] = -2^{3-k}\prod_{i=1}^{3}(n_a^i l_a^i) \ , \tag{10}$$

$$I_{aO6} = [\Pi_a][\Pi_{O6}] = 2^{3-k}(-l_a^1 l_a^2 l_a^3 + l_a^1 n_a^2 n_a^3 + n_a^1 l_a^2 n_a^3 + n_a^1 n_a^2 l_a^3) \ , \tag{11}$$

where $[\Pi_{O6}] = [\Pi_{\Omega R}] + [\Pi_{\Omega R\omega}] + [\Pi_{\Omega R\theta\omega}] + [\Pi_{\Omega R\theta}]$ is the sum of O6-plane homology three-cycles wrapped by the four O6-planes, and $k = \beta_1 + \beta_2 + \beta_3$ is the total number of tilted two-tori. For future convenience, we shall define $n_a^i l_b^i \pm n_b^i l_a^i$ as the intersection factor from the $i-$th two-torus.

The general spectrum of D6-branes intersecting at generic angles, which is valid for both rectangular and tilted two-tori, is given in Table I. And the 4-dimensional $N = 1$ supersymmetric models from Type IIA orientifolds with intersecting D6-branes are mainly constrained in two aspects: RR tadpole cancellation (section 2.1) and $N = 1$ supersymmetry in four dimensions (section 2.2).

A. Tadpole Cancellation Conditions

As sources of RR fields, D6-branes and orientifold O6-planes are required to satisfy the Gauss law in a compact space, *i.e.*, the total RR charges of D6-branes and O6-planes must vanish since the RR field flux lines are conserved. The RR tadpole cancellation conditions are

$$\sum_a N_a[\Pi_a] + \sum_a N_a\,[\Pi_{a'}] - 4[\Pi_{O6}] = 0\ , \tag{12}$$

where the last contribution comes from the O6-planes which have -4 RR charges in the D6-brane charge unit.

To simplify the notation, let us define the products of wrapping numbers

$$\begin{array}{rl} A_a \equiv -n_a^1 n_a^2 n_a^3, & B_a \equiv n_a^1 l_a^2 l_a^3, \quad C_a \equiv l_a^1 n_a^2 l_a^3, \quad D_a \equiv l_a^1 l_a^2 n_a^3, \\ \tilde{A}_a \equiv -l_a^1 l_a^2 l_a^3, & \tilde{B}_a \equiv l_a^1 n_a^2 n_a^3, \quad \tilde{C}_a \equiv n_a^1 l_a^2 n_a^3, \quad \tilde{D}_a \equiv n_a^1 n_a^2 l_a^3. \end{array} \tag{13}$$

To cancel the RR tadpoles, we can also introduce an arbitrary number of D6-branes wrapping cycles along the orientifold planes, the so called "filler branes", which contribute to the tadpole conditions but trivially satisfy the supersymmetry conditions. Thus, the tadpole conditions are

$$\begin{array}{c} -\,2^k N^{(1)} + \sum_a N_a A_a = -2^k N^{(2)} + \sum_a N_a B_a = \\ -2^k N^{(3)} + \sum_a N_a C_a = -2^k N^{(4)} + \sum_a N_a D_a = -16, \end{array} \tag{14}$$

where $2N^{(i)}$ are the number of filler branes wrapping along the i-th O6-plane which is defined in Table II.

The tadpole cancellation conditions directly lead to the $SU(N_a)^3$ cubic non-Abelian anomaly cancellation [15, 16, 27]. And the cancellation of U(1) mixed gauge and gravitational anomaly or $[SU(N_a)]^2 U(1)$ gauge anomaly can be achieved by Green-Schwarz mechanism mediated by untwisted RR fields [15, 16, 27].

B. Conditions for 4-Dimensional $N = 1$ Supersymmetric D6-Brane

The four-dimensional $N = 1$ supersymmetric models require that 1/4 supercharges from ten-dimensional Type I T-dual be preserved, *i.e.*, these 1/4 supercharges should survive two projections: the orientation projection of the intersecting D6-branes, and the $Z_2 \times Z_2$ orbifold projection on the background manifold. Analysis shows that, the 4-dimensional $N = 1$ supersymmetry (SUSY) can be preserved by the orientation projection if and only

TABLE II: Wrapping numbers of the four O6-planes.

Orientifold Action	O6-Plane	$(n^1,l^1)\times(n^2,l^2)\times(n^3,l^3)$
ΩR	1	$(2^{\beta_1},0)\times(2^{\beta_2},0)\times(2^{\beta_3},0)$
$\Omega R\omega$	2	$(2^{\beta_1},0)\times(0,-2^{\beta_2})\times(0,2^{\beta_3})$
$\Omega R\theta\omega$	3	$(0,-2^{\beta_1})\times(2^{\beta_2},0)\times(0,2^{\beta_3})$
$\Omega R\theta$	4	$(0,-2^{\beta_1})\times(0,2^{\beta_2})\times(2^{\beta_3},0)$

if the rotation angle of any D6-brane with respect to the orientifold-plane is an element of $SU(3)$[9], or in other words, $\theta_1+\theta_2+\theta_3=0 \bmod 2\pi$, where θ_i is the angle between the $D6$-brane and the orientifold-plane in the i-th two-torus. Meanwhile, this 4-dimensional $N=1$ supersymmetry will automatically survive the $Z_2\times Z_2$ orbifold projection. The SUSY conditions can therefore be written as [29]

$$x_A\tilde{A}_a+x_B\tilde{B}_a+x_C\tilde{C}_a+x_D\tilde{D}_a=0,$$

$$A_a/x_A+B_a/x_B+C_a/x_C+D_a/x_D<0, \tag{15}$$

where $x_A=\lambda,\ x_B=\lambda 2^{\beta_2+\beta_3}/\chi_2\chi_3,\ x_C=\lambda 2^{\beta_1+\beta_3}/\chi_1\chi_3,\ x_D=\lambda 2^{\beta_1+\beta 2}/\chi_1\chi_2$, and $\chi_i=R_i^2/R_i^1$ are the complex structure moduli. The positive parameter λ has been introduced to put all the variables A, B, C, D on an equal footing. Based on these SUSY conditions, all possible D6-brane configurations preserving 4-dimensional $N=1$ supersymmetry can be classified into three types:

(1) Filler brane with the same wrapping numbers as one of the O6-planes in Table II. It corresponds to the USp group. And among coefficients A, B, C and D, one and only one of them is non-zero and negative. If the filler brane has non-zero A, B, C or D, we refer to the USp group as the A-, B-, C- or D-type USp group, respectively.

(2) Z-type D6-brane which contains one zero wrapping number. Among A, B, C and D, two are negative and two are zero.

(3) NZ-type D6-brane which contains no zero wrapping number. Among A, B, C and D, three are negative and the other one is positive. Based on which one is positive, these NZ-type branes are defined as the A-, B-, C- and D-type NZ brane. Each type can have two forms of the wrapping numbers which are defined as

$$A1:(-,-)\times(+,+)\times(+,+),\ \ A2:(-,+)\times(-,+)\times(-,+); \tag{16}$$

$$B1:(+,-)\times(+,+)\times(+,+),\ B2:(+,+)\times(-,+)\times(-,+); \tag{17}$$
$$C1:(+,+)\times(+,-)\times(+,+),\ C2:(-,+)\times(+,+)\times(-,+); \tag{18}$$
$$D1:(+,+)\times(+,+)\times(+,-),\ D2:(-,+)\times(-,+)\times(+,+). \tag{19}$$

In the following, we'll call the Z-type and NZ-type D6-branes as U-branes since they carry $U(n)$ gauge symmetry.

III. T-DUALITY SYMMETRY AND ITS VARIATIONS

T-duality relates equivalent models, and thus employing this symmetry can simplify significantly the search for the inequivalent models. In this section, we shall study the action of the T-dualities (and its variants). These symmetries correspond to two types: one is general and can be applied to any D6-brane model building (type I) and the other one is special and only effective in the Pati-Salam model building (type II). Our philosophy is: consider only one model for each equivalent class characterized by these T-dualities.

Before discussing the T-duality, we point out that: (1) Two models are equivalent if their three two-tori and the corresponding wrapping numbers for all the D6-branes are related by an element of permutation group S_3 which acts on three two-tori. This is a trivial fact. (2) Two D6-brane configurations are equivalent if their wrapping numbers on two arbitrary two-tori have the same magnitude but opposite sign, and their wrapping numbers on the third two-torus are the same. We call it as the D6-brane Sign Equivalent Principle (DSEP). In the following, we discuss type I and type II T-dualities separately:

Type I T-duality: T-duality transformation happens on two two-tori simultaneously, for example, the j-th and k-th two-tori

$$(n_x^j, l_x^j) \longrightarrow (-l_x^j, n_x^j),\ (n_x^k, l_x^k) \longrightarrow (l_x^k, -n_x^k)\ , \tag{20}$$

where x runs over all stacks of D6-branes in the model. It doesn't change anything about the D-brane model except that it makes an interchange among the four pairs of the products of wrapping numbers $(A, \tilde{A})$, $(B, \tilde{B})$, $(C, \tilde{C})$ and $(D, \tilde{D})$, which indicates that the particle spectra are invariant under this transformation while the complex structure moduli may not be.

Without loss of generality, we assume that $j = 2$ and $k = 3$. In this case, the interchange will take place between A and B pairs, and C and D pairs

$$(A, \tilde{A}) \leftrightarrow (B, \tilde{B}),\ (C, \tilde{C}) \leftrightarrow (D, \tilde{D}). \tag{21}$$

And the corresponding transformations of moduli parameters are

$$x'_A = x_B,\ x'_B = x_A,\ x'_C = x_D,\ x'_D = x_C, \tag{22}$$

After these transformations, we obtain the new complex structure moduli and radii of T^6

$$\chi'_1 = 2^{\beta_1}\sqrt{\frac{x_A x_B}{x_C x_D}} = \chi_1, \tag{23}$$

$$\chi'_2 = 2^{\beta_2}\sqrt{\frac{x_D x_B}{x_A x_C}} = 2^{2\beta_2}(\chi_2)^{-1}, \tag{24}$$

$$\chi'_3 = 2^{\beta_3}\sqrt{\frac{x_C x_B}{x_A x_D}} = 2^{2\beta_3}(\chi_3)^{-1}, \tag{25}$$

$$(R_1^1)' = R_1^1,\ (R_1^2)' = R_1^2, \tag{26}$$

$$(R_2^1)' = \frac{2^{-\beta_2}M_s^2}{R_2^1},\ (R_2^2)' = \frac{2^{\beta_2}M_s^2}{R_2^2}, \tag{27}$$

$$(R_3^1)' = \frac{2^{-\beta_3}M_s^2}{R_3^1},\ (R_3^2)' = \frac{2^{\beta_3}M_s^2}{R_3^2}, \tag{28}$$

where M_s is the string scale.

Sometimes, it will be more convenient if we combine this T-duality with the trivial two two-tori interchange $T_j \leftrightarrow T_k$ where the transformations of the wrapping numbers and $\beta_{j,k}$ are

$$n_x^j \leftrightarrow n_x^k,\ l_x^j \leftrightarrow l_x^k, \tag{29}$$

$$\beta_j \leftrightarrow \beta_k. \tag{30}$$

Thus, under this extended T-duality, the wrapping numbers and $\beta_{j,k}$ transform as

$$(n_x^j, l_x^j) \longrightarrow (l_x^k, -n_x^k),\ (n_x^k, l_x^k) \longrightarrow (-l_x^j, n_x^j), \tag{31}$$

$$\beta_j \leftrightarrow \beta_k. \tag{32}$$

Still for $j = 2$ and $k = 3$, only A and B pairs are interchanged under this T-duality

$$(A, \tilde{A}) \leftrightarrow (B, \tilde{B}), \tag{33}$$

and the corresponding transformations of moduli parameters are

$$x'_A = x_B, x'_B = x_A, x'_C = x_C, x'_D = x_D. \tag{34}$$

In this case, the new complex structure moduli and radii are

$$\chi'_1 = 2^{\beta_1}\sqrt{\frac{x_A x_B}{x_C x_D}} = \chi_1, \tag{35}$$

$$\chi'_2 = 2^{\beta_3}\sqrt{\frac{x_B x_C}{x_A x_D}} = 2^{2\beta_3}\chi_3^{-1}, \tag{36}$$

$$\chi'_3 = 2^{\beta_2}\sqrt{\frac{x_B x_D}{x_A x_C}} = 2^{2\beta_2}\chi_2^{-1}, \tag{37}$$

$$(R_1^1)' = R_1^1,\ (R_1^2)' = R_1^2, \tag{38}$$

$$(R_2^1)' = \frac{2^{-\beta_3} M_s^2}{R_3^1},\ (R_2^2)' = \frac{2^{\beta_3} M_s^2}{R_3^2}, \tag{39}$$

$$(R_3^1)' = \frac{2^{-\beta_2} M_s^2}{R_2^1},\ (R_3^2)' = \frac{2^{\beta_2} M_s^2}{R_2^2}. \tag{40}$$

One pair of the models related by this extended type I T-duality have been shown in Table VII and Table VIII in the Appendix up to DSEP on the first two two-tori of b stack of D6-branes. Since this extended T-duality only interchanges two pairs of the products of wrapping numbers, if all two-tori are rectangular or tilted, all models characterized by the permutations of these four parameter pairs will be T-dual to each other. By the way, for the case where two two-tori are rectangular or tilted, this conclusion is not valid if the rectangular and tilted two-tori have been fixed.

As a remark, in this kind of D6-brane models, the Yang-Mills gauge coupling g_{YM}^x for x-stack of D6-branes at string scale is [33, 39]

$$(g_{YM}^x)^2 = \frac{\sqrt{8\pi} M_s}{M_{Pl}} \frac{1}{\prod_{i=1}^{3} \sqrt{\left(n_x^i\right)^2 \chi_i^{-1} + \left(2^{-\beta_i} l_x^i\right)^2 \chi_i}}\ , \tag{41}$$

where M_{Pl} is the 4-dimensional Planck scale. So the gauge coupling is invariant under the T-duality and its variation. This is a typical property of T-duality(e.g. see [40]).

Type II T-duality: Under it, the transformations of the wrapping numbers for any stacks of D6-branes in the model are

$$n_x^i \to -n_x^i,\ l_x^i \to l_x^i,\ n_x^j \leftrightarrow l_x^j,\ n_x^k \leftrightarrow l_x^k, \tag{42}$$

where $i \neq j \neq k$, and x runs over all D6-branes in the model. Comparing with the general type I T-duality, besides the interchanges among $(A, \tilde{A})$, $(B, \tilde{B})$, $(C, \tilde{C})$ and $(D, \tilde{D})$, the signs of $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ and $\tilde{D}$ are also changed, which lead to the sign changes of all intersection numbers. On the other hand, for the models in our model construction, we require that

$$I_{ab} + I_{ab'} = 3,\ I_{ac} = -3,\ I_{ac'} = 0, \tag{43}$$

which will be discussed in detail in the next Section. Obviously, there is only one sign difference between the intersection numbers of I_{ab} and I_{ac} if $I_{ab} = 3$. By combining with

$$b \leftrightarrow c, \tag{44}$$

therefore, we may get one equivalent model satisfying our requirements. For this type of T-duality, the moduli and radii will obey the same transformation rules as those in the type

I T-duality. But, unlike the type I, the quantum numbers for $SU(2)_L$ and $SU(2)_R$ in the particle spectrum and two gauge couplings at string scale will be interchanged due to $b \leftrightarrow c$. Models in Table VII and Table IX are such a pair of examples related by type II T-duality up to DSEP on b and c stacks of D6-branes.

If $I_{ab} = 1$ or 2, which can be achieved only when $n_a^i l_b^i = 0$ or $n_b^i l_a^i = 0$ are satisfied on two two-tori, these intersection numbers become

$$I_{ab'} = 3, \ I_{ac'} = -1 \text{ or } -2, \tag{45}$$

under the type II T-duality transformation. Therefore, if we relax the intersection number requirement to

$$I_{ab} + I_{ab'} = 3, \ I_{ac} = -3, \ I_{ac'} = 0, \tag{46}$$

and

$$I_{ab} = 3, \ I_{ab'} = 0, \ I_{ac} + I_{ac'} = -3, \tag{47}$$

we only have to consider one case if the D6-brane wrapping numbers in two setups can be related by Eq. (42): the derivations and conclusions in the first setup can be applied to the second one as well.

By combining with type I T-duality and DSEP, we obtain an variation of type II T-duality. Under it, the transformations of the wrapping numbers for any stacks of D6-branes in the model are

$$\begin{aligned} & l_x^1 \to -l_x^1, \ l_x^2 \to -l_x^2, \ l_x^3 \to -l_x^3, \\ & b \leftrightarrow c, \end{aligned} \tag{48}$$

where x runs over all D6-branes in the model. Since the transformations in the first line of the above equations only change the signs of $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ and $\tilde{D}$, the moduli and radii are invariant.

IV. SUPERSYMMETRIC PATI-SALAM MODELS AND GAUGE SYMMETRY BREAKING VIA D6-BRANE SPLITTINGS

To build the SM or SM-like models in the intersecting D6-brane scenarios, besides $U(3)_C$ and $U(2)_L$ stacks of branes, we need at least two extra $U(1)$ gauge symmetries for both SUSY and non-SUSY versions due to the quantum number of the right-handed electron [16, 27, 28, 29]. One ($U(1)_L$) is lepton number symmetry, and the other one ($U(1)_{I_{3R}}$) is like the third component of right-handed weak isospin. Then, the hypercharge is obtained via

$$Q_Y = Q_{I_{3R}} + \frac{Q_B - Q_L}{2} \ , \tag{49}$$

where $U(1)_B$ arises from the overall $U(1)$ in $U(3)_C$. Meanwhile, to forbid the gauge field of $U(1)_{I_{3R}}$ to obtain a mass via $B \wedge F$ couplings, the $U(1)_{I_{3R}}$ can only come from the non-Abelian part of $U(2)_R$ or USp gauge symmetry. In this case, the $U(1)$ gauge symmetry, which comes from a non-Abelian symmetry, is generically anomaly free and its gauge field is massless. Similarly, to generate the anomaly-free $U(1)_{B-L}$ symmetry, the $U(1)_L$ should come from non-Abelian group. Considering that the $U(1)_L$ stack should be parallel to the $U(3)_C$ stack on at least one two-tori, we generate it by splitting branes from one $U(4)$ stack, resulting in the $U(3)_C$ stack at the same time.

In the previous supersymmetric SM constructions [27, 28], $U(1)_{I_{3R}}$ arises from the stack of D6-branes on the top of orientifold, *i.e.*, from the USp group. These models have at least 8 pairs of the SM Higgs doublets, and generically there exist two additional anomaly free $U(1)$ gauge symmetries. They could be in principle spontaneously broken via Higgs mechanism by the scalar components of the chiral superfields with the quantum numbers of the right-handed neutrinos, however they break D-flatness conditions, and thus supersymmetry, so the scale of symmetry breaking should be near the electroweak scale. On the other hand, there were typically no candidates, preserving the D-flatness and F-flatness conditions, and that could in turn break these gauge symmetries at an intermediate scale.

Therefore, we focus on Pati-Salam model where $U(1)_{I_{3R}}$ arises from the $U(2)_R$ symmetry. Failing to find interesting models with $SU(2)_L$ from the D6-branes on the top of O6-plane, we would like to construct the supersymmetric $SU(4)_C \times SU(2)_L \times SU(2)_R$ models from three stacks of D6-branes that are not on the top of orientifold planes. We will show that the Pati-Salam gauge symmetry can be broken down to $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ via D6-brane splittings, and then down to the SM gauge symmetry via Higgs mechanism where the Higgs particles come from a $N = 2$ subsector. In particular, in our models, we do not have any extra anomaly free U(1) gauge symmetry at electroweak scale which was a generic problem in previous constructions [27, 28].

Suppose we have three stacks of D6-branes, a, b, and c with number of D6-branes 8, 4, and 4. So, a, b, and c stacks give us the gauge symmetry $U(4)_C$, $U(2)_L$ and $U(2)_R$, respectively. The anomalies from three $U(1)$s are cancelled by the Green-Schwarz mechanism, and the gauge fields of these $U(1)$s obtain masses via the linear $B \wedge F$ couplings. So, the effective gauge symmetry is $SU(4)_C \times SU(2)_L \times SU(2)_R$. In addition, we require that the intersection numbers satisfy

$$I_{ab} + I_{ab'} \;=\; 3 \;, \tag{50}$$

$$I_{ac} \;=\; -3 \;,\; I_{ac'} \;=\; 0 \;. \tag{51}$$

The conditions $I_{ab} + I_{ab'} = 3$ and $I_{ac} = -3$ give us three families of the SM fermions with quantum numbers $(\mathbf{4}, \mathbf{2}, \mathbf{1})$ and $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ under $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetry.

$I_{ac'} = 0$ implies that a stack of D6-branes is parallel to the orientifold (ΩR) image c' of the c stack of D6-branes along at least one tow-torus, for example, the third two-torus. Then, there are open strings which stretch between the a and c' stacks of D6-branes. If the minimal distance squared $Z^2_{(ac')}$ (in $1/M_s$ units) between these two stacks of D6-branes on the third two-torus is small, *i.e.*, the minimal length squared of the stretched string is small, we have the light scalars with squared-masses $Z^2_{(ab')}/(4\pi^2\alpha')$ from the NS sector, and the light fermions with the same masses from the R sector [15, 16, 39]. These scalars and fermions form the 4-dimensional $N = 2$ hypermultiplets, so, we obtain the $I^{(2)}_{ac'}$ (the intersection numbers for a and c' stacks on the first two two-tori) vector-pairs of the chiral multiplets with quantum numbers $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ and $(\mathbf{4}, \mathbf{1}, \mathbf{2})$. These particles are the Higgs fields needed to break the Pati-Salam gauge symmetry down to the SM gauge symmetry. In particular, these particles are massless if $Z^2_{(ac')} = 0$. By the way, the intersection numbers $I_{ac} = 0$ and $I_{ac'} = -3$ are equivalent to $I_{ac} = -3$ and $I_{ac'} = 0$ due to the symmetry transformation $c \leftrightarrow c'$.

In order to break the gauge symmetry, we split the a stack of D6-branes into a_1 and a_2 stacks with 6 and 2 D6-branes, respectively. The $U(4)_C$ gauge symmetry is broken down to the $U(3) \times U(1)$. Let us assume that the numbers of symmetric and anti-symmetric representations for $SU(4)_C$ are respectively $n^a_{\square\square}$ and $n^a_{\substack{\square\\\square}}$, similar convention for $SU(2)_L$ and $SU(2)_R$. After splitting, the gauge fields and three multiplets in adjoint representation for $SU(4)_C$ are broken down to the gauge fields and three multiplets in adjoint representations for $SU(3)_C \times U(1)_{B-L}$, respectively. The $n^a_{\square\square}$ and $n^a_{\substack{\square\\\square}}$ multiplets in symmetric and anti-symmetric representations for $SU(4)_C$ are broken down to the $n^a_{\square\square}$ and $n^a_{\substack{\square\\\square}}$ multiplets in symmetric and anti-symmetric representations for $SU(3)_C$, and $n^a_{\square\square}$ multiplets in symmetric representation for $U(1)_{B-L}$. However, there are $I_{a_1 a_2'}$ new fields with quantum number $(\mathbf{3}, -\mathbf{1})$ under $SU(3)_C \times U(1)_{B-L}$ from the open strings at the intersections of D6-brane stacks a_1 and a_2'. The rest of the particle spectrum is the same. Moreover, we can show that the anomaly free gauge symmetry from a_1 and a_2 stacks of D6-branes is $SU(3)_C \times U(1)_{B-L}$, which is the subgroup of $SU(4)_C$.

Furthermore, we split the c stack of D6-branes into c_1 and c_2 stacks with 2 D6-branes for each one. Similarly, the gauge fields and three multiplets in adjoint representation for $SU(2)_R$ are broken down to respectively the gauge fields and three multiplets in adjoint representation for $U(1)_{I_{3R}}$, The $n^c_{\square\square}$ multiplets in symmetric representation for $SU(2)_R$ are broken down to the $n^c_{\square\square}$ multiplets in symmetric representation for $U(1)_{I_{3R}}$, while the $n^c_{\substack{\square\\\square}}$ multiplets in anti-symmetric representation for $SU(2)_R$ are gone. In addition, there are $I_{c_1 c_2'}$ new fields which are neutral under $U(1)_{I_{3R}}$ from the open strings at the intersections of D6-brane stacks c_1 and c_2'. And the rest of the particle spectrum is the same. Moreover, the

anomaly free gauge symmetry from c_1 and c_2 stacks of D6-branes is $U(1)_{I_{3R}}$, which is the subgroup of $SU(2)_R$.

After D6-brane splittings, we obtain that the gauge symmetry is $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$. To break this gauge symmetry down to the SM gauge symmetry, we assume that the minimal distance squared $Z^2_{(a_2 c_1')}$ between the a_2 and c_1' stacks of D6-branes on the third two-torus is very small, then, we obtain $I^{(2)}_{a_2 c_1'}$ (the intersection numbers for a_2 and c_1' stacks on the first two two-tori) pairs of chiral multiplets with quantum numbers $(\mathbf{1}, \mathbf{1}, \mathbf{-1}, \mathbf{1/2})$ and $(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{-1/2})$ under $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ from the light open string states which stretch between the a_2 and c_1' stacks of D6-branes. These particles can break the $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ down to the SM gauge symmetry and keep the D- and F-flatness because their quantum numbers are the same as those for the right-handed neutrino and its complex conjugate. Especially, these particles are massless if $Z^2_{(a_2 c_1')} = 0$. In summary, the complete symmetry breaking chains are

$$
\begin{aligned}
SU(4) \times SU(2)_L \times SU(2)_R \quad & \overrightarrow{a \to a_1 + a_2} \; SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\
& \overrightarrow{c \to c_1 + c_2} \; SU(3)_C \times SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L} \\
& \overrightarrow{\text{Higgs Mechanism}} \; SU(3)_C \times SU(2)_L \times U(1)_Y \; .
\end{aligned} \tag{52}
$$

The dynamical supersymmetry breaking in D6-brane models from Type IIA orientifolds has been addressed in [33]. In the D6-brane models, there are some filler branes carrying USp groups which are confining, and thus could allow for gaugino condensation, supersymmetry breaking and moduli stabilization.

The gauge kinetic function for a generic stack x of D6-branes is of the form (see, e.g., [33]):

$$
f_x = \tfrac{\mathbf{1}}{\mathbf{4}} \left[n_x^1 n_x^2 n_x^3 S - \left(\sum_{i=1}^{3} 2^{-\beta_j - \beta_k} n_x^i l_x^j l_x^k U^i \right) \right] , \tag{53}
$$

where the real parts of dilaton S and moduli U^i are

$$
\mathrm{Re}(S) = \frac{M_s^3 R_1^1 R_1^2 R_1^3}{2\pi g_s} \;, \tag{54}
$$

$$
\mathrm{Re}(U^i) = \mathrm{Re}(S) \; \chi_j \chi_k \;, \tag{55}
$$

where $i \neq j \neq k$, and g_s is the string coupling. Also, the Kähler potential is

$$
K = -\ln(S + \bar{S}) - \sum_{I=1}^{3} \ln(U^I + \bar{U}^I). \tag{56}
$$

In our models, three stacks of D6-branes with $U(4)_C \times U(2)_L \times U(2)_R$ gauge symmetry generically fix the complex structure moduli χ_1, χ_2 and χ_3 due to supersymmetry conditions.

So, there is only one independent modulus field. To stabilize the modulus, we need at least two USp groups which are confining, *i.e.*, their β functions are negative, and thus allow for gaugino condensations [41, 42, 43]. Suppose there are $2N^{(i)}$ filler branes which are on top of i-th O6-plane and carry $USp(N^{(i)})$ group. Its beta function is

$$\begin{aligned}
\beta_i^g &= -3(\frac{N^{(i)}}{2}+1) + 2|I_{ai}| + |I_{bi}| + |I_{ci}| + 3(\frac{N^{(i)}}{2}-1) \\
&= -6 + 2|I_{ai}| + |I_{bi}| + |I_{ci}| \ ,
\end{aligned} \tag{57}$$

where $3(N^{(i)}/2-1)$ is a contribution from three multiplets in the anti-symmetric representation of the USp branes. The negative β functions of USp groups give strong constraints on the intersection numbers of the associated filler branes and the observable branes, and thus constrain the number of allowed models.

If supersymmetry turns out to be broken due to the gaugino condensations, the supersymmetry breaking will be mediated via gauge interactions because the gravity mediated supersymmetry breaking is much smaller. So, the supersymmetry CP problem can be solved in our models. In this paper, we will neither study the stabilization of this complex structure modulus (or for that reason also Kähler moduli, which could enter the gauge coupling corrections due to the one-loop threshold corrections) due to gaugino condensation, nor the issue of supersymmetry breaking and postpone this for further study. However, since we do eventually want to address these issues, we confine our search only to models with at least two USp gauge group factors with negative β functions.

V. SYSTEMATIC SEARCH FOR SUPERSYMMETRIC PATI-SALAM MODELS

The basic properties for the models that we want to construct are given in Section III. Let us summarize them here. There are three stacks of D6-branes, a, b, and c with number of D6-branes 8, 4, and 4, which give us the gauge symmetry $U(4)_C$, $U(2)_L$ and $U(2)_R$, respectively. We require that their intersection numbers satisfy Eqs. (50−51). In addition, to stabilize the modulus and possibly break the supersymmetry, we require that at least two USp groups in the hidden sector have negative β functions.

Our searching strategy is the following: first, analytically exclude most of the parameter space for the D6-brane wrapping numbers which can not give the models with above properties, and then scan the rest parameter space by employing a computer program. If no two-torus is tilted, we can not have the particle spectra with odd families of the SM fermions. So, there are three possibilities: one tilted two-torus, two tilted two-tori, and three tilted two-tori. The complete searching shows no-go for the last two possibilities. As for the first one, all solutions are tabulated in the Appendix. The detailed discussions are given in the

following three subsections. People only interested in phenomenology may safely skip these parts.

A. One Tilted Two-Torus

Without loss of generality, let us suppose that the third two-torus is tilted. Then, we may consider the two cases where the a stack of D6-branes is of NZ- and Z-type, which are characterized by no and one zero wrapping number respectively.

Case I: NZ-type a Stack of D6-branes with One Negative Wrapping Number

For NZ-type a stack of D6-branes, among A_a, B_a, C_a and D_a, there is at least one equal to -1 in order to avoid the tadpole cancellation condition (TCC) violations. Due to T-duality, we may assume that $D_a = -1$. Obviously, the setup $I_{ab} = 1$ or 2 can not be realized in this case. As for $|l_a^3|$ there are only two possible absolute values (ABS): 1 and 3, because the third wrapping number pairs (WNP) should be responsible for the even factors of both $2^k I_{ac} = \pm 6$ and $2^k I_{ac'} = 0$ $(k = 1)$. We shall discuss this case according to the number of negative wrapping numbers for a-brane because this number can not be larger than 3 due to the D6-brane Sign Equivalent Principle (DSEP). Next, let us take a look at the case with one negative wrapping number first.

Since $D_a = -1$, the minus sign can only come from l_a^1, l_a^2 and n_a^3. Noticing that SUSY conditions can not be satisfied if $n_a^3 < 0$, we set $l_a^1 < 0$ without loss of generality. If $l_a^3 = 3$, the a-brane will be

$$(+,-1) \times (+,1) \times (1,3)\ . \tag{58}$$

For the general intersection numbers $2^k I_{ax} = \pm 3 \times 2^k$ and $I'_{ax} = 0$, in order to generate the associated even factor requires at least one set of WNPs from the tilted two-torus which satisfy $|n_a^i| = |n_x^i|$, $|l_a^i| = |l_x^i|$, and $n_a^i l_a^i = -n_x^i l_x^i = \pm 3$ or ± 1 while the co-prime conditions are also implemented. As a result, the 3rd WNP of the C-type brane should be $(\pm 1, \mp 3)$, and thus one C2-brane is needed for CTCC (Tadpole Cancellation Condition related to C) requirement

$$(-,+) \times (+,+) \times (-1,3)\ . \tag{59}$$

As for the third brane, if it is of A2-, B2- or D1-type,

$$(-,+) \times (-,+) \times (-1,3)\ , \tag{60}$$

$$(+,+) \times (-,+) \times (-1,3)\ , \tag{61}$$

$$(+,+) \times (+,+) \times (1,-3)\ , \tag{62}$$

there will be a problematic intersection factor generated from the second or the first WNP. This factor has a ABS larger than 1, which will lead to $2^k I_{ax} > 6$. If the third brane is of C2-type also ATCC and DTCC require all wrapping numbers of a-, b- and c-branes have unit ABS, which obviously is forbidden in our model building. Therefore, the third brane is of Z-type and can not provide extra positive Tadpole Charges. A direct result for this is that $|n_a^1 n_a^2| = 1$ and $|n_C^1 n_C^2| \leq 2$, $|l_C^1 l_C^2| \leq 2$ due to ATCC and DTCC. But, any one of $|n_C^1 n_C^2|$ and $|l_C^1 l_C^2|$ can not be equal to 1 to avoid vanishing I_{aC}, which may lead to $n_3^3 = 0$ due to ATCC and DTCC again. This is impossible. Therefore, there is no solution while $l_a^3 = 3$.

If $l_a^3 = 1$, the most general form for the a-brane is

$$(L_1, -1) \times (L_2, 1) \times (1, 1) \ , \tag{63}$$

where L_1 and L_2 are positive. If $L_2 > 2$ for a-brane, no matter what value L_1 has, both A2- and C2-type branes are needed

$$(-, +1) \times (-, +1) \times (-1, +1) \ , \tag{64}$$

$$(-, +1) \times (+, +1) \times (-1, +1) \ . \tag{65}$$

It is easy to see that the 2nd WNP of the A2-brane will contribute an intersection factor larger than L_2 or 3. Since we have another even factor from the third WNP, this may lead to $2^k I_{aA} > 6$. Therefore, there is no solution in this case.

For $L_2 = 2$, both A2- and C2-branes are still required. If both of the 2nd and 3rd branes are of Z-type, to avoid ATCC and CTCC violation, we must have $n_2^2 = n_3^2 = 0$. Then the second WNPs of these branes will yield an even intersection factor because L_2 is even, thus yielding even SM families. So, the 2nd and 3rd branes can not be Z-type at the same time. If one of them is of NZ-type, the other one must be NZ-type also because the combination of a-brane and the second one will necessarily violate ATCC and/or CTCC. As a result, b- and c-branes will be

$$(-(L_1 \pm 1), +1) \times (-1, +1) \times (-1, +1) \ , \tag{66}$$

$$(-, +1) \times (+, +1) \times (-1, +1) \ . \tag{67}$$

It is obvious that the factor 3 for $2^k I_{aA}$ should be generated from the second WNP, thus we have $|n_A^2 l_A^2| = 1$ here. ATCC can not be satisfied while $L_1 \geq 2$; as for $L_1 = 1$, ATCC requires $|n_C^1 n_C^2| \leq 2$, which will forbid that I_{aC} be a multiple of 3. Therefore, the only possible solution for a-brane in this case is

$$(L, -1) \times (1, 1) \times (1, 1) \ . \tag{68}$$

Let us consider the case where $L > 2$ first. In this case, one A2- brane is required, and C- and D-type USp groups can not appear in the hidden sector since they are not asymptotically free according to Eq. (57). Noticing the intersection factor 3 of $2^k I_{aA}$ should be generated by the second WNP, we have two kinds of possible solutions

$$(L,-1)\times(1,1)\times(1,1)\ , \tag{69}$$
$$(-(L\pm 1),1)\times(-2,1)\times(-1,1)\ , \tag{70}$$

and

$$(L,-1)\times(1,1)\times(1,1)\ , \tag{71}$$
$$(-(L\pm 1),1)\times(-1,2)\times(-1,1)\ , \tag{72}$$
$$(+,+)\times(-1,0)\times(-1,1)\ . \tag{73}$$

For the former, no matter what the third brane is, the combination of a- and A-branes yield no solution for moduli consistent with supersymmetry; for the latter, obviously, a problematic intersection factor larger than 3 comes out from the first WNPs of a- and the third branes.

Finally, let us prove that the ABSs of b- and c-branes' wrapping number can not be larger than 8 for $L \leq 2$. If they are NZ-type branes, this conclusion is obvious since for each WNP only one component's ABS can be larger than 2 to avoid TCC violation. Thus the possible largest ABS of wrapping numbers should be less than $2L+3=7$. For Z-type brane, the possible ABS larger than 8 should come from the WNP without zero wrapping number. Meanwhile, the corresponding two-torus is untilted. Let us focus on such a WNP. Suppose this brane contributes the non-vanishing X- and Y-type tadpole charges. If X- and Y- do not match with the types of a- and the 3rd brane, due to the same reason applied to NZ-type brane, the wrapping numbers' ABS of this brane still can not be larger than 7. If $X=B$ and the 3rd brane is Y-type brane, we shall have $l_Z^1 n_Z^2 = 0$. For $l_Z^1=0$, in order to avoid the problematic factor generated from the 2nd WNPs, we must have $|n_Z^2 l_Z^2| = 2$ since n_Z^2 and l_Z^2 have different signs due to SUSY conditions. For $n_Z^2=0$, given $|B_Z| \leq 2L+4 \leq 8$ or $|n_Z^1| \leq 2L+4 \leq 8$, l_Z^1 can not be larger than 7 due to the co-prime conditions for both $L=1$ and 2.

Case II: NZ-type a Stack of D6-branes with Two and Three Negative Wrapping Numbers

For the case with two minus signs, if they are from different two-tori, the only possible setup for a-brane due to SUSY conditions is

$$(L_1,1)\times(L_2,-1)\times(1,-)\ , \tag{74}$$

which corresponds to case I by the type II T-duality inference. Therefore, we need not consider this case any more.

If the two minus signs come from the same two-torus, based on DSEP, let us suppose that they are from the first two-torus. While $l_a^3 = 3$, the a-brane will be of the form

$$(-L_1, -1) \times (L_2, 1) \times (1, 3) \ , \tag{75}$$

and both B2- and C2-branes

$$(+, +) \times (-, +) \times (-1, 3) \ , \tag{76}$$
$$(-, +) \times (+, +) \times (-1, 3) \ , \tag{77}$$

are required. Obviously, a problematic intersection factor larger than 1 will be generated for $2^k I_{aC}$. Therefore, there is no solution at this time.

If $l_a^3 = 1$, the a-brane will be of the form

$$(-L_1, -1) \times (L_2, 1) \times (1, 1) \ . \tag{78}$$

For $L_1 \geq 2$ and $L_2 > 2$ or $L_1 > 2$ and $L_2 \geq 2$, B2- and C2-branes are needed

$$(+, +1) \times (-, +1) \times (-1, +1) \ , \tag{79}$$
$$(-, +1) \times (+, +1) \times (-1, +1) \ , \tag{80}$$

where at least one problematic intersection factor will be contributed by the 2nd WNP of B2-brane or the 1st WNP of C2-brane. Therefore the only possible solution for a-brane will be

$$(-2, -1) \times (2, 1) \times (1, 1) \ , \tag{81}$$

and

$$(-L, -1) \times (1, 1) \times (1, 1) \ . \tag{82}$$

Noticing that the second setup can be transformed to the setup in Case I through type II T-duality, we only need to consider the first one.

For the first setup, B2- and C2-branes are required and the only possible solutions are

$$(-2, -1) \times (2, 1) \times (1, 1) \ , \tag{83}$$
$$(-1, 1) \times (2 \pm 1, 1) \times (-1, 1) \ , \tag{84}$$
$$(2 \mp 1, 1) \times (-1, 1) \times (-1, 1) \ , \tag{85}$$

Regretfully, these solutions are excluded because there are no moduli solutions for supersymmetric D6-branes configuration. And thus, the case with NZ-type a stack of D6-branes

with two negative wrapping numbers should be ruled out. For the case with three negative wrapping numbers, due to SUSY conditions the only possible setup for a-brane is

$$(L_1, -1) \times (L_2, -1) \times (-1, 1) \ , \tag{86}$$

which obviously corresponds to the case with two minus signs according to the type II T-duality variation. Thus, this case can be excluded, too.

Case III: Z-type a Stack of D6-branes

Now let us consider the Z-type a-brane. According to the type I T-duality and its extended version, there are two possible setups for a-brane: $A_a = B_a = 0$ and $A_a = D_a = 0$. The latter can not give the required intersection numbers with b- and c-branes. If $|n_a^3 l_a^3| = 3$, according to SUSY conditions, the a-brane has four kinds of possible setups

$$(0, -1) \times (L_1, L_2) \times (1, 3) \ , \tag{87}$$

$$(0, -1) \times (L_1, L_2) \times (3, 1) \ , \tag{88}$$

$$(0, 1) \times (L_1, -L_2) \times (1, -3) \ , \tag{89}$$

$$(0, 1) \times (L_1, -L_2) \times (3, -1) \ . \tag{90}$$

The 1st and 2nd setups correspond to the 4th and 3rd ones due to type I T-duality, respectively, and the 1st one also corresponds to the 3rd one due to type II T-duality. Therefore we only need to consider the 1st one. For the 1st one, one C2-brane is required and it has the form

$$(-1, +) \times (+, 1) \times (-1, 3) \ . \tag{91}$$

In order to avoid BTCC and DTCC violation (since the third brane can provide at most one kind of positive tadpole charge), we must have $l_C^2 = 1$. In addition, since $n_3^1 \neq 0$ due to $I_{a3} \neq 0$, in order to avoid the BTCC violation we have to require that the third brane satisfy $l_3^2 = 0$

$$(-, -) \times (+, 0) \times (-1, +) \ . \tag{92}$$

If $l_3^2 \neq 0$, we must have $l_3^3 = 3$ and thus the third brane is of B-type

$$(+, +) \times (-, +) \times (-1, 3) \ . \tag{93}$$

A problematic intersection factor will arise from the second WNP. Eq. (92) implies that the third brane is the b-brane and thus c-brane is of C-type. Note that here $I_{ac} + I_{ac'} = -3$,

which is T-dual (type II) to the 3rd possible setup of a-brane with $I_{ab}+I_{ab'}=3$, can not be achieved here. Since DTCC requires $l_a^2=1$ and $|l_c^1|\le 2$, $I_{ac}=-3$ will yield $n_a^2=n_c^2+1>0$, and thus CTCC can not be satisfied. So, there is no solution while $l_a^3=3$.

For the case with $|n_a^3 l_a^3|=1$, based on the type I T-duality and SUSY conditions, we can have only a-brane of the form:

$$(0,-1)\times(L_1>0,L_2>0)\times(1,1)\ . \tag{94}$$

Meanwhile, due to type II T-duality, we only need consider the case $L_1>L_2$. Let us prove first that there is no solution while $L_1>2$. If $L_1>2$, one C2-brane is required, but the third brane must be of Z-type. The reason is that, if the third brane is of A2- or B2-type

$$(-,+)\times(-,+)\times(-1,1)\ , \tag{95}$$

$$(+,+)\times(-,+)\times(-1,1)\ , \tag{96}$$

there is a problematic intersection factor larger than L_1 generated from the second WNPs. If it is C2- or D1-brane,

$$(-,+)\times(+,+)\times(-1,1)\ , \tag{97}$$

$$(-,-)\times(+,+)\times(-1,1)\ , \tag{98}$$

$n_3^2=l_3^2L_1+1$ or $l_3^2L_1+3$ is forbidden in order to avoid the ATCC violation. Therefore, I_{ab} and I_{ac} have the same signs if both of them are of C2- or D1-type, which is not allowed. Thus $L_2=1$ is required.

Let us return to the C2-brane. If the intersection factor 3 for I_{aC} comes from the first two-torus, one additional A2-brane is needed because $n_C^1=-3$ and $n_C^2\ge 2$ has led to ATCC violation. Therefore, there is no solution in this case. If the factor 3 comes from the second two-torus, l_C^2 can not be larger than 2, otherwise, DTCC can not be satisfied. The only two possible solution for a- and C-type branes will be

$$(0,-1)\times(3,1)\times(1,1)\ , \tag{99}$$

$$(-1,+)\times(3,2)\times(-1,1)\ , \tag{100}$$

and

$$(0,-1)\times(L,1)\times(1,1)\ , \tag{101}$$

$$(-1,+)\times(L-3,1)\times(-1,1)\ . \tag{102}$$

For the first possible solution, because DTC (ATC, BTC, DTC and DTC denote the A-, B-, C- and D-type tadpole charges, respectively.) is already filled, the 3rd brane should satisfy $l_3^1 l_3^2=0$ and have the form

$$(-1,l_3^1)\times(1,l_3^2)\times(-1,1)\ , \tag{103}$$

where $A_3 = -1$ is required by ATCC. Obviously, the intersection factor 3 can not be generated for I_{a3}. For the second possible solution, ATCC requires $4 \leq L \leq 7$. To avoid the CTCC violation, $l_C^1 > 2$ is necessary, which, however, will lead to the DTCC violation. As a result, there is no solution if $L_1 > 2$.

For $L = 1, 2$, a solution is possible. Here we shall prove that the wrapping numbers of b- and c-branes can not be larger than 8. For a NZ-type brane, the only possible wrapping number with its ABS larger than 3 is l^1. $|n^2|$ and $|l^2|$ must be smaller than $2L+3$ since the smaller one still can not be larger than 2. $|l^1|$ can be larger than 3 only when the NZ-brane is of C- or D-type. Correspondingly, the third brane should be of D- or C-type. Since the components of the 3rd WNPs for both branes have different signs, the two branes should be of C2- and D1-types

$$(0, -1) \times (L, 1) \times (1, 1) \ , \tag{104}$$

$$(+, l_C^1 < 0) \times (+, +) \times (1, -1) \ , \tag{105}$$

$$(+, +) \times (+, +) \times (1, -1) \ . \tag{106}$$

Due to ATCC and BTCC, one of I_{aC} and I_{aD} can not be a multiple of 3. For a Z-type brane, as we discussed before, the possible wrapping number with its ABS larger than 3 can only come from an untilted two-torus and the corresponding WNP contains no zero wrapping number. If this WNP is from the second torus, SUSY conditions require that its two components have different signs and thus their ABS can not be larger than 2 to avoid a problematic intersection factor. If this WNP is from the first two-torus, the only possible large wrapping number is l_Z^1. If $l_Z^1 > 2$, the third brane is of C-type or D-type. For the C-type case, given that ATCC and DTCC require $|A_3| \leq 3$ and $|D_3| \leq 2$, respectively, CTCC will yield $|C_Z|$ or $|l_Z^1| \leq |A_3| \times |D_3| + 4 - 2 = 8$. As a result, the largest ABS of these wrapping numbers is less than 9. We shall have the same situation for D-type 3rd brane.

B. Two Tilted Two-Tori

Similar to the previous Subsection, we consider the NZ-type a stack of D6-branes first and then turn to the Z-type one. Here we assume that the second and the third two-tori are tilted.

Case I: NZ-type a Stack of D6-branes with One Negative Wrapping Number

For the a-brane of NZ-type, let us still suppose $D = -1$ in accordance with T-duality.

Then this brane has two forms

$$(L_1,-1)\times(L_2,1)\times(1,L_3)\ , \tag{107}$$

and

$$(L_1,1)\times(L_2,-1)\times(1,L_3)\ , \tag{108}$$

where L_1, L_2, L_3 are positive, and L_2, L_3 are odd. According to the requirement of moduli stabilization for gauge symmetries in the hidden sector, there is at most one with its ABS larger than 2 among all wrapping numbers of a-brane, which implies three possibilities: (1)$L_3 = 1$, $L_1 \le 2$ and $L_2 \ge 3$; (2)$L_2 = 1$, $L_1 \le 2$ and $L_3 \ge 3$; (3)$L_2 = L_3 = 1$ and $L_1 \ge 2$.

Now let us consider the first form of a-brane and set $L_2 \ge 3$

$$(L_1,-1)\times(L_2,1)\times(1,1)\ , \tag{109}$$

then the other two branes are of A- and C-types, respectively, and have $D_A = D_C = -1$ to avoid DTCC violation. Meanwhile, to avoid a problematic intersection factor, the C-brane should be of C2-type:

$$(-L_1\pm1<0,+1)\times(L_2\pm6>0,+1)\times(-1,1)\ , \tag{110}$$

where the intersection factor 3 of I_{aC} can not come from the first WNP. If it does, we shall have $n_C^1 = -(L_1+3)$ because of $0 < L_1 \le 2$. The C-type USp group is no longer asymptotically free, and then we do not have enough USp groups in the hidden sector to stabilize the modulus. Now let us consider the A-type brane. For A1-brane, the 0 intersection factor for I_{aA} can not be generated. The A-brane, therefore, should be of A2-type. The factor 3 of I_{aA} comes from the second WNP, and thus the brane will be

$$(-L_1\pm1<0,+1)\times(L_2\pm6<0,+1)\times(-1,1)\ . \tag{111}$$

Under this setup, we always have $n_A^1 = n_C^1$ no matter L_1 is equal to 1 or 2. This is obvious for $L_1 = 1$. If $L_1 = 2$, both n_A^1 and n_C^1 can not be equal to 3. Otherwise, C-type USp group is not asymptotically free again. So we shall have $n_A^1 = n_C^1 = -1$. On the other hand, because of $n_C^2 > 0$ and $n_A^2 < 0$, we must have $n_C^2 = L_2 + 6$ and $n_A^2 = L_2 - 6$, and thus $|n_C^2| - |n_A^2| = 2L_2 > 0$. Due to $L_2 \ge 3$, the ATCC can not be satisfied for any value of L_2. As a result, we have $L_2 = 1$ for the first setup of a-brane.

If $L_3 \ge 3$, the a-brane has the form

$$(L_1,-1)\times(1,1)\times(1,L_3)\ , \tag{112}$$

and then one C-type brane is required due to CTCC. At this time A- and D-type USp groups are not asymptotically free due to the large value of L_3. If the C-brane is of C1-type, the intersection factor 3 for I_{aC} should come from the 1st WNP and the only possible solutions for the a- and C1-branes are

$$(L_1, -1) \times (1, 1) \times (1, L_3) \ , \tag{113}$$

$$(n_C^1 > 0, l_C^1 > 0) \times (1, -1) \times (1, L_3 + 2) \ , \tag{114}$$

where $L_1 n_C^1 l_C^1 = 2$ to provide the factor 3 for I_{aC}, n_C^3 can not be equal to 2 to keep $ATC = DTC = 0$, and l_C^3 can not be equal to $L_3 - 2$ to avoid CTCC violation. Now it is easy to check that there is no moduli solution while $L_1 = 2$ or $l_C^1 = 2$, and B-type USp group can not be generated in the hidden sector while $n_C^1 = 2$. If this is a C2-brane, the intersection factor 3 will come from the third WNP, which means $L_3 = 3$, thus we shall have $n_C^2 l_C^2 \geq 3$. Noticing that the third brane is of NZ-type no matter which one is larger, it is easy to check that there are no enough asymptotically free USp groups available in the hidden sector.

Finally, let us consider the last kind of setup of a-brane

$$(L_1 \geq 2, -1) \times (1, 1) \times (1, 1) \ . \tag{115}$$

For $L_1 \geq 3$, one brane of A-type is required. This is also true for $L_1 = 2$. Otherwise, to avoid one A-type brane, the fact that ATC is full will lead to $n^1 n^2 n^3 = 0$ for the other two branes. n^1 can not be equal to zero to avoid an even problematic intersection factor. If $n^2 = 0$, the intersection factor 3 comes from the first WNP and this brane has the form

$$(-2l^1 \pm 3 < 0, l^1) \times (0, 2) \times (-1, 1) \ . \tag{116}$$

To avoid the DTCC violation, the third brane must have $n_3^3 = 0$ and $|l_3^3| = 2$. This indicates that no enough USp groups are available in the hidden sector now. As a result, one A-type brane is definitely necessary while $L_1 \geq 2$.

This A-type brane is a A2-brane to get $I_{aA'} = 0$. If the intersection factor 3 of I_{aA} comes from the first WNP, the possible solution for this brane is

$$(-(l_A^1 L_1 \pm 3) < 0, l_A^1) \times (-1, 1) \times (-1, 1) \ , \tag{117}$$

which gives out no moduli solutions by combining with a-brane setup. If the factor 3 is given out by the second WNPs, we must have $L_1 = 2$, in order to preserve at least two asymptotically free USp groups in the hidden sector. Thus the possible solution for a- and A-type brane will be

$$(2, -1) \times (1, 1) \times (1, 1) \ , \tag{118}$$

$$(-1, 1) \times (-(6 - l_A^2) < 0, l_A^2) \times (-1, 1) \ , \tag{119}$$

which yield no moduli solutions while $l_A^2 < 3$. If $l_A^2 = 4$ or 5, DTCC requires that the third brane be of D-type, which necessarily leads to less than two asymptotically free USp groups available in the hidden sector.

As for the second form of a-brane in this case,

$$(L_1, 1) \times (L_2, -1) \times (1, L_3) \ , \tag{120}$$

where $L_1 \geq 2$, $L_2 \geq 3$ and $L_3 \geq 3$, the derivation is similar. For the case $L_1 \geq 2$, both A- and B-type branes are required. This is obvious for $L_1 \geq 3$. For $L_1 = 2$, if there is no A- and B-type branes, the fact that ATC and BTC are filled requires $n_2^1 = n_3^1 = 0$, in order to avoid the ATCC and BTCC violation, which in turn will lead to a problematic even factor from the first WNPs since L_1 is even. These two necessary NZ-type branes have $D_A = D_B = -1$ due to DTCC. If the B-brane is of B1-type, the factor 3 for I_{aB} comes from the first WNP and thus we have $n_a^1 = 2$ and $L_2 = 1$. Then, the possible solutions for a-brane and the B-brane will be

$$(2, 1) \times (1, -1) \times (1, 1) \ , \tag{121}$$

$$(1, -1) \times (1, 1) \times (1, 3) \ , \tag{122}$$

with $L_3 = 1$ required by the BTCC because of $B_B \leq L_3 + 2$. It is easy to check that under this setup, A1-brane can not obtain the intersection factor 3 to avoid the BTCC and CTCC violation, and A2-brane can not yield the moduli solution by combining with the B-type brane.

Now let us consider the combination of B2- and A1-branes. L_2 can not be larger than 3, otherwise, one problematic factor will be generated for I_{aA} from the second WNPs. If $L_2 = 3$, B- and D-type USp groups are no longer asymptotically free. To ensure C-type USp group is asymptotically free, we must have $L_1 = 2$. Then the only possible solutions are

$$(2, 1) \times (3, -1) \times (1, 1) \ , \tag{123}$$

$$(+, 1) \times (-, 1) \times (-1, 1) \ , \tag{124}$$

$$(-1, -1) \times (3, 1) \times (1, 3) \ . \tag{125}$$

But, due to BTCC, n_B^1 can not be smaller than 3, *i.e.*, C-type USp group is still not asymptotically free. So we must have $L_2 = 1$. As for L_3, we shall have the same situation if it is larger than 1. So, the last possible solutions for a-, B- and A-branes are

$$(L_1, 1) \times (1, -1) \times (1, 1) \ , \tag{126}$$

$$(L_1 \pm 3 > 0, 1) \times (-3, 1) \times (-1, 1) \ , \tag{127}$$

$$(-(L_1 \pm 3) < 0, -1) \times (1, 1) \times (1, 3) \ , \tag{128}$$

where $l_A^3 = -n_B^2 = 3$ is because the intersection factor 3s of I_{aB} and I_{aA} cannot come from the tilted two-tori. This is straightforward to see. For example, if the intersection factor 3 of I_{aB} comes from the 2nd WNP, we must have $n_B^2 = -7$ and then the ATCC will require $n_A^1 \geq 7$. This is not allowed since B-, C- and D-type USp groups are not asymptotically free in this case. For this set of possible solutions, ATCC requires $L_1 \leq 3$, then we shall have $n_B^1 = -n_A^1 = L_1 + 3$; BTCC is violated. As for the B2-A2 combination, the factor 3 for I_{aA} comes from the first WNP, and thus we have $L_3 = 1$. The possible solutions are

$$(2,1) \times (1,-1) \times (1,1) \ , \tag{129}$$

$$(n_B^1 > 0, 1) \times (3,-1) \times (1,-1) \ , \tag{130}$$

$$(-1,1) \times (-3,1) \times (-1,1) \ , \tag{131}$$

where $L_2 = 1$ is required by ATCC because of $A_A \leq L_2 + 2$, and $n_B^2 \leq 3$ is required by CTCC. Now it is easy to see that ATCC will forbid I_{aB} obtain the intersection factor 3. As a result, for $L_1 \geq 2$, there is no solution.

We now turn to $L_1 = 1$ case. If $L_2 \geq 3$, B- and D-type USp groups can not appear in the hidden sector and a-brane will have the form

$$(1,1) \times (L_2,-1) \times (1,1) \ . \tag{132}$$

One A-type brane is required. For A1-brane, the factor 3 for I_{aA} will be contributed by the second WNPs and thus we have $n_A^3 l_A^3 = 3$. If $n_A^3 = 3$, the third brane is of D1-type since D2-type brane can not generate the zero factor for $I_{aD'}$. Because no extra positive BTC is available, we must have $B_a = B_A = B_D = 1$. Then the only possible solutions are

$$(1,1) \times (3,-1) \times (1,1) \ , \tag{133}$$

$$(-1,-2) \times (3,1) \times (3,1) \ , \tag{134}$$

$$(1,2) \times (3,1) \times (1,-1) \ , \tag{135}$$

which, however, is excluded by CTCC. If $l_A^3 = 3$, the third brane is of B-type and it should be B2-brane since the problematic intersection factor can not be avoided for B1-brane. Due to $BTC = DTC = 0$, the possible solutions are

$$(1,1) \times (3,-1) \times (1,1) \ , \tag{136}$$

$$(-2,-1) \times (3,1) \times (1,3) \ , \tag{137}$$

$$(4,1) \times (1,-1) \times (1,-1) \ , \tag{138}$$

which means that the C-type USp group is not asymptotically free, either.

If the A-brane is of A2-type, the factor 3 of I_{aA} will be provided by the first WNPs, indicating $|n_A^1 l_A^1| = 2$. Meanwhile, l_A^2 can not be larger than 1 to satisfy $BTC = DTC = 0$

while the 3rd brane is also counted in. Then the possible solution for a- and A2-brane will be

$$(1,1)\times(3,-1)\times(1,1)\ , \tag{139}$$

$$(-1,2)\times(n_A^2<0,1)\times(-1,1)\ , \tag{140}$$

where $l_A^1=2$ is because there is no moduli solutions for $l_A^1=1$. But, no proper value is available for n_A^2: $|n_A^2|=1$ will lead to ATCC or DTCC violation when the third brane is included, and $|n_A^2|=5$ will make C-type USp group unavailable since the third brane must be of Z-type to satisfy $DTC=0$. As a result, there is no solution for $L_2\geq 3$. As for the case where $L_3\geq 3$, due to the extended type I T-duality, it is dual to the case where $L_2\geq 3$ according to Eq. (32), and thus all above conclusion can be applied directly to this case.

At last, let us recall that the ABS of all branes' wrapping numbers are less than 8 under the survived two setups of a-brane

$$(1,-1)\times(1,1)\times(1,1)\ , \tag{141}$$

$$(1,1)\times(1,-1)\times(1,1)\ . \tag{142}$$

For the former, the proof is exactly the same as in the case of one-tilted two-torus. For the latter, the only difference happens where $n_2^1=0$ and the third brane is of D-type. In that case, ATCC, BTCC and CTCC will impose on the 3rd brane the constraint $D_3\leq 5$. Combining this constraint and CTCC, it implies that the wrapping numbers of the 2nd brane can not be larger than 8 in this case, either.

Case II: NZ-type a Stack of D6-branes with Two and Three Negative Wrapping Numbers

In this case, the forms of a-brane allowed by the SUSY conditions include

$$(-L_1,-1)\times(L_2,1)\times(1,L_3)\ ,\quad (L_1,-1)\times(L_2,1)\times(1,-L_3)\ , \tag{143}$$

$$(L_1,1)\times(L_2,-1)\times(1,-L_3)\ ,\ (-L_1,1)\times(L_2,-1)\times(1,-L_3)\ . \tag{144}$$

The first case corresponds to

$$(L_1,-1)\times(L_3,1)\times(1,L_2)\ , \tag{145}$$

by a T-duality combination of the Eqs. (32) and (48). As for the last three cases, they correspond to

$$(L_1,1)\times(L_2,-1)\times(1,L_3)\ , \tag{146}$$

$$(L_1,-1)\times(L_2,1)\times(1,L_3)\ , \tag{147}$$

$$(-L_1,-1)\times(L_2,1)\times(1,L_3)\ , \tag{148}$$

respectively, according to the extended type II T-duality Eq. (48). Therefore, these setups are also excluded due to T-duality.

Case III: Z-type a Stack of D6-branes

Let us consider the case where the a-brane is of Z-type. Suppose that the zero wrapping number comes from the untilted two-torus, then due to T-duality, we can assume $A_a = B_a = 0$ or $n_a^1 = 0$ and this a-brane has the form

$$(0, -1) \times (+, +) \times (+, +) \ . \tag{149}$$

For the four unknown positive wrapping numbers, at least one of them is larger than 2; if not, all of them are equal to 1. Without loss of generality, we may suppose n_a^2 is such a wrapping number, then it must be true that $l_a^2 = n_a^3 = l_a^3 = 1$. Before verifying this, we point out first that the other two branes should be of C- and D-type only if one of l_a^2, n_a^3 and l_a^3 is not equal to 1. This is obvious if l_a^3 is not the largest one among these three wrapping numbers. If l_a^3 is the largest one, it should be larger than 2, and then A- and B-type USp groups are no longer asymptotically free. C-type brane is definitely required due to $n_a^2 \geq 3$ here. So, to preserve the D-type USp group in the hidden sector, we must have one D-type brane to provide the extra positive DTC. Next, we shall discuss these cases separately.

For $n_a^3 > 1$, the intersection factor 3s of I_{aC} and I_{aD} will come from the tilted two-tori, which means, among n_C^2, n_C^3, n_D^2 and n_D^3, there are two whose ABS is equal to 3. This will lead to ATCC violation. Thus we must have $n_a^3 = 1$. For l_a^3, it can not be larger than 3, otherwise, we shall have $n_C^2 = n_D^2 = 3$ to generate the intersection factor 3 and ATCC will be violated again. If $l_a^3 = 3$, the only possible solutions are

$$(0, -1) \times (3, 1) \times (1, 3) \ , \tag{150}$$

$$(1, l_2^1) \times (3, -1) \times (1, 1) \ , \tag{151}$$

$$(1, l_3^1) \times (1, 1) \times (1, -3) \ . \tag{152}$$

Obviously, no matter what values l_2^1 and l_3^1 take, CTCC and DTCC can not be satisfied at the same time. So l_a^3 must be equal to 1 also. For $l_a^2 > 1$, suppose that $n_a^2 > l_a^2$ due to T-duality, the intersection factor 3, no matter for I_{aC} or I_{aD}, can not come from the first WNP. For example, if I_{aC}'s does, ATCC and BTCC will require $A_D = B_D = -1$ since now we have $A_C = B_C = -3$. Then the D-type brane can not obtain the intersection factor 3. As a result, the only possible solutions will be

$$(0, -1) \times (n_a^2, l_a^2) \times (1, 1) \ , \tag{153}$$

$$(-1, l_C^1 > 0) \times (n_C^2 > 0, l_C^2 > 0) \times (-1, 1) \ , \tag{154}$$

$$(1, l_D^1 > 0) \times (n_D^2 > 0, l_D^2 > 0) \times (1, -1) \ . \tag{155}$$

Since $|n_C^2 l_C^2| \leq 3$ and $|n_D^2 l_D^2| \leq 3$, it is not hard to check one by one that the intersection factor 3 of I_{aC} and I_{aD} can not be generated from the second WNPs at the same time. So, the a-brane can not have two wrapping numbers with their ABSes larger than 1 at the same time. The last possibility is then

$$(0, -1) \times (n_a^2 > 0, 1) \times (1, 1) \ . \tag{156}$$

For the last possible setup of a-brane, if $n_a^2 \geq 3$, one C-type brane is required. For a C1-brane, the intersection factor 3 for I_{aC} comes from the second WNP, and the possible solutions of a- and C1-branes are

$$(0, -1) \times (3, 1) \times (1, 1) \ , \tag{157}$$

$$(1, l_C^1 > 0) \times (3, -1) \times (1, 3) \ , \tag{158}$$

where n_C^3 must be equal to 1 in order to avoid ATCC and/or DTCC violation after the third brane is included. Meanwhile, to generate the intersection factor 3 of I_{a3}, $|n_3^1 n_3^2| \geq 3$ or $|A_3| \geq 3$ can not be avoided, so the last brane should be of A-type. Noticing that $|n_3^1| = 3$ is not allowed by BTCC, we have $|n_3^2| = 3$, which necessarily leads to no enough asymptotically free USp groups in the hidden sector. For a C2-brane, the intersection factor 3 of I_{aC} can not come from the first WNPs. If it does, one extra D- or A-type brane is required. For a D-type brane, $A_D = B_D = 1$ will forbid I_{aD} to obtain the intersection factor 3. For an A1-brane, BTCC and DTCC require $|n_A^3 l_A^3| = 1$ which will lead to $I_{aA} = 0$. For an A2-brane, we have $B_A = D_A = D_C = -1$ and thus the intersection factor 3 of I_{aA} can only come from the second WNPs. The only possible solutions are

$$(0, -1) \times (3, 1) \times (1, 1) \ , \tag{159}$$

$$(-3, 1) \times (n_C^2, 1) \times (-1, 1) \ , \tag{160}$$

$$(-1, 1) \times (-3, 1) \times (-1, 1) \ . \tag{161}$$

Obviously n_C^2 has no solution to satisfy CTCC and ATCC at the same time. As a result, the intersection factor 3 of I_{aC} has to come from the 2nd WNPs and the only possible solutions for a- and C2-brane will be

$$(0, -1) \times (n_a^2 \geq 3, 1) \times (1, 1) \ , \tag{162}$$

$$(-1, l_C^1 > 0) \times (n_a^2 \pm 6 > 0, 1) \times (-1, 1) \ , \tag{163}$$

where l_C^2 can not be larger than 3 to avoid the requirement of additional B- and D-type branes. If it is equal to 3, there is no solution for n_a^2 satisfying the co-prime condition. It

can not be equal to 2 or 3 also to avoid the requirement of additional A- and D-type branes are required at the same time. If it is equal to 2, we have $n_C^2 \geq 4$ and two additional ATC and DTC are required together. As for the n_C^2, if it is equal to n_a^2+6, the third brane is of A-type, and we have $B_A=-3$ and $D_A=-1$, or $l_A^2=1$ and $|n_A^1 l_A^3|=3$. If $l_A^3=3$, this is a A1-brane and $n_A^2=n_a^2\pm 6>0$. It is easy to check that in either case, ATCC and CTCC can not be satisfied at the same time. If $l_A^3=1$, this is a A2-brane and we have $n_A^1=3$. At this time, a problematic intersection factor for I_{aA} will be generated from the 2nd WNPs. If $n_C^2=n_a^2-6$, CTCC requires $l_C^1>2$ and thus the 3rd brane is of D-type. Meanwhile, we have $B_D=n_D^1 l_D^2 l_D^3=-3$. For a D2-type brane, one problematic intersection factor for I_{aD} will be generated by the 2nd WNPs due to $n_a^2\geq 7$. For a D1-type brane, $l_D^3=-1$; if $l_D^2=3$, $n_D^1=1$ and there is no solution for n_D^2 satisfying the co-prime condition. Therefore, the possible solutions are

$$(0,-1)\times(n_a^2\geq 7,1)\times(1,1)\ , \tag{164}$$

$$(-1,l_C^1>0)\times(1,1)\times(-1,1)\ , \tag{165}$$

$$(3,l_D^1>0)\times(1,1)\times(1,-1)\ , \tag{166}$$

regretfully, there is no solution for l_C^1 and l_D^1 satisfying CTCC and DTCC at the same time. Based on the above analysis, we can declare the no-go theorem for $n_a^2>1$. As for the case where $n_a^2=1$, following the same logic as applied in the case of one-tilted two-torus, it is easy to see that no wrapping number can have ABS larger than 8.

If the zero wrapping number comes from one of the tilted two-tori, due to type I T-duality and its extended version again, we may take $n_a^2=0$ and thus $|l_a^2|=2$. The other two branes will be B2- and D1-branes. If they are of Z-type, their second WNPs have to satisfy $|n^2|=2$ and $l^2=0$, which will yield a problematic intersection factor 4 since the third WNPs have yielded another even factor. Furthermore, they must be of B2- and D1-types because B1- and D2-types branes can not provide the zero factor to $I_{aB'}$ and $I_{aD'}$. The possible solutions then are

$$(n_a^1>0,l_a^1>0)\times(0,-2)\times(1,1)\ , \tag{167}$$

$$(+,+)\times(1,-)\times(1,-1)\ , \tag{168}$$

$$(+,+)\times(1,+)\times(1,-1)\ , \tag{169}$$

up to the inferential Type II T-duality. Here the intersection factor 3 for I_{aB} and I_{aD} can not come from the 2nd WNPs. If one of them does obtain the intersection factor 3 from the 2nd WNPs, the other one will be forbidden to obtain the intersection factor 3 due to ATCC and CTCC constraints. The factor 3 can not come from the 3rd WNPs also since one of ATCC and CTCC will be violated only if n_a^3 or l_a^3 is equal to 3. Now let us consider the last

case – they are from the 1st WNPs. To generate these intersection factor 3, $n_a^1 = l_a^1 = 1$ is not allowed, since $n_B^1 l_B^1 \leq 3$ and $n_D^1 l_D^1 \leq 3$ due to ATCC and CTCC. Without loss of generality, let us suppose $n_a^1 < l_a^1$. Then according to $I_{aB} = -I_{aD}$, we have

$$(l_B^1 + l_D^1) n_a^1 = (n_B^1 + n_D^1) l_a^1 \ . \tag{170}$$

Given $l_B^1 + l_D^1 \leq 4$, $n_B^1 + n_D^1 \leq 4$ and the co-prime conditions, (n_a^1, l_a^1) have three possible solutions $(3, 4)$, $(2, 3)$ and $(1, 2)$. Now it is not hard to figure out one by one that the intersection factor 3s can not be generated from the 1st WNPs for I_{aB} and I_{aD} at the same time. As a result, there is no solution in this case.

C. Three Tilted Two-Tori

If all two-tori are tilted, for one set of definite values arranged to $\{A, B, C, D\}$, models characterized by any permutations of them actually are T-dual to each other. If a-brane is of NZ-type, considering that there is only one wrapping number L with its ABS larger than 1 to preserve enough asymptotically free USp groups, we only need to consider two kinds of possible setups for a-brane: $\{A = -B = L, C = D = -1\}$ and $\{A = B = -L, C = -D = 1\}$. And the C- and D-type USp groups are not asymptotically free.

For the former, without loss of generality, we may suppose that a-brane is of A1-type up to the inferential type II T-duality,

$$(-L, -1) \times (1, 1) \times (1, 1) \ . \tag{171}$$

Then one of the other two branes must be of B-type. In the case of a B1-brane, the intersection factor 3 will come from the 1st WNP and $L = n_B^1 = n_B^2 l_B^2 = n_B^3 l_B^3 = 3$. So, no enough asymptotically free USp groups can be preserved. In the case of a B2-brane, still for the sake of enough asymptotically free USp groups in the hidden sector, the intersection factor 3 must come from the first WNPs. Meanwhile, due to CTCC and DTCC, we have $l_B^1 = 1$ or $l_B^1 = l_B^2 = l_B^3 = -n_B^2 = -n_B^3 = 1$ where there are no moduli solutions for a- and B2- branes.

For the latter, we may suppose that a-brane is a C1-brane

$$(L, 1) \times (1, -1) \times (1, 1) \ . \tag{172}$$

Then the other two branes must be of A- and B-types and $D_A = D_B = -1$ since a-brane is of C-type. In order to keep enough asymptotically free USp groups, the intersection factor 3 must come from the first WNPs and only the combinations of A1- and B2-branes, and

A2- and B1-branes are allowed

$$(L,1)\times(1,-1)\times(1,1)~, \tag{173}$$
$$(-(L\pm 6)<0,-1)\times(1,1)\times(1,3)~, \tag{174}$$
$$(L\pm 6>0,1)\times(-3,1)\times(-1,1)~, \tag{175}$$

and

$$(L,1)\times(1,-1)\times(1,1)~, \tag{176}$$
$$(-(L\pm 6)<0,1)\times(-3,1)\times(-1,1)~, \tag{177}$$
$$(L\pm 6>0,-1)\times(1,1)\times(1,3)~. \tag{178}$$

For the first one, ATCC and BTCC can not be satisfied only if $L>1$; for the second one, the combination of A2- and B1-branes gives no moduli solutions.

As for the case where $L=1$, it is easy to see that the ABS of all branes' wrapping numbers are less than 8. Meanwhile, if a-brane has one vanishing wrapping number, supposing $(0,-2)$ for the second WNP of a-brane, we know that the intersection factor 3 of I_{aB} and I_{aD} cannot come from the 2nd WNPs. Meanwhile, they can not come from the same WNPs also due to ATCC and CTCC. So, the only possible solutions are

$$(1,3)\times(0,-2)\times(3,1)~, \tag{179}$$
$$(1,-3)\times(1,\pm|l_2^2|)\times(1,1)~, \tag{180}$$
$$(1,1)\times(1,\pm|l_3^2|)\times(3,-1)~. \tag{181}$$

However, in this case the BTCC or DTCC is violated. Thus, no solution exists!

D. Preliminary Phenomenological Features of the Models

After analytically excluding most of parameter space for wrapping numbers, we wrote a computer program to scan the rest parameter space. The results indicate that no model is available for the cases with two and three tilted two-tori. For the case with one-tilted two-torus, we obtain 11 inequivalent models which can be specified by two classes.

The first class include Tables VII−XVII, which correspond to the models that do not possess the gauge coupling unification for $SU(2)_L$ and $SU(2)_R$ at the string scale and whose Higgs doublets are generated by the intersections of b- and c-stacks of branes.

For the second class, which includes Tables XVIII and XIX, they have the $SU(2)_L$ and $SU(2)_R$ gauge coupling unification at the string scale. Because the b-stack branes for these models are parallel to both c-stack branes and their images on one of the three two-tori,

their Higgs doublet pairs come from the massless open string states in a $N = 2$ subsector and form vector-like pairs.

For all of these models, except I-Z-7, I-Z-8, and I-Z-9, the number of the pairs of Higgs doublets is less than 9, which could make them phenomenologically interesting. In particular, there are two pairs of Higgs doublets in model I-NZ-1a. As an example, the chiral particle spectra for models I-NZ-1a, I-Z-5 and I-Z-10 are presented in Tables III-V, respectively. For model I-NZ-1a, it has 2 pairs of Higgs doublets and two confining hidden sector gauge group factors. Model I-Z-5 has 3 pairs of Higgs doublets and three confining gauge group factors. Model I-Z-10 has the gauge coupling unification of $SU(2)_L$ and $SU(2)_R$ at the string scale and 4 confining gauge group factors. These models thus possess potentially phenomenologically attractive freatures.

TABLE III: The chiral spectrum in the open string sector of model I-NZ-1a

I-NZ-1a	$SU(4) \times SU(2)_L \times SU(2)_R$ $\times USp(2) \times USp(4) \times USp(2)$	Q_4	Q_{2L}	Q_{2R}	Q_{em}	$B-L$	Field
ab	$3 \times (4, \overline{2}, 1, 1, 1, 1)$	1	-1	0	$-\frac{1}{3},\ \frac{2}{3},\ -1,\ 0$	$\frac{1}{3},\ -1$	Q_L, L_L
ac	$3 \times (\overline{4}, 1, 2, 1, 1, 1)$	-1	0	1	$\frac{1}{3},\ -\frac{2}{3},\ 1,\ 0$	$-\frac{1}{3},\ 1$	Q_R, L_R
bc'	$2 \times (1, 2, 2, 1, 1, 1)$	0	1	1	$1,\ 0,\ 0,\ -1$	0	H
$a1$	$1 \times (4, 1, 1, 2, 1, 1)$	1	0	0	$\frac{1}{6},\ -\frac{1}{2}$	$\frac{1}{3},\ -1$	
$a2$	$1 \times (\overline{4}, 1, 1, 1, 4, 1)$	-1	0	0	$-\frac{1}{6},\ \frac{1}{2}$	$-\frac{1}{3},\ 1$	
$a3$	$1 \times (4, 1, 1, 1, 1, \overline{2})$	1	0	0	$\frac{1}{6},\ -\frac{1}{2}$	$\frac{1}{3},\ -1$	
$b1$	$2 \times (1, \overline{2}, 1, 2, 1, 1)$	0	-1	0	$\mp\frac{1}{2}$	0	
$b2$	$1 \times (1, 2, 1, 1, 4, 1)$	0	1	0	$\pm\frac{1}{2}$	0	
$c3$	$4 \times (1, 1, 2, 1, 1, 2)$	0	0	1	$\pm\frac{1}{2}$	0	
$a_{\Yasymm}$	$4 \times (6, 1, 1, 1, 1, 1)$	2	0	0	$-\frac{1}{3}, 1$	$-\frac{2}{3}, 2$	
$b_{\Ysymm}$	$1 \times (1, 3, 1, 1, 1, 1)$	0	2	0	$0, \pm 1$	0	
$b_{\overline{\Yasymm}}$	$1 \times (1, \overline{1}, 1, 1, 1, 1)$	0	-2	0	0	0	
$c_{\overline{\Ysymm}}$	$3 \times (1, 1, \overline{3}, 1, 1, 1)$	0	0	-2	$0, \pm 1$	0	
$c_{\Yasymm}$	$3 \times (1, 1, 1, 1, 1, 1)$	0	0	2	0	0	

Hidden sector gauge groups with negative beta functions have a potential to be confining. In these cases, the gaugino condensations generate the non-perturbative effective superpotential. The minimization of this supergravity potential determines the ground state which can stabilize the dilaton and complex structure toroidal moduli, and in some cases breaks

TABLE IV: The chiral spectrum in the open string sector of model I-Z-5

I-Z-5	$SU(4)\times SU(2)_L\times SU(2)_R\times USp(2)^3$	Q_4	Q_{2L}	Q_{2R}	Q_{em}	$B-L$	Field
ab	$3\times(4,\overline{2},1,1,1,1)$	1	-1	0	$-\frac{1}{3},\ \frac{2}{3},\ -1,\ 0$	$\frac{1}{3},\ -1$	Q_L, L_L
ac	$3\times(\overline{4},1,2,1,1,1)$	-1	0	1	$\frac{1}{3},\ -\frac{2}{3},\ 1,\ 0$	$-\frac{1}{3},\ 1$	Q_R, L_R
bc'	$3\times(1,\overline{2},\overline{2},1,1,1)$	0	-1	-1	$-1,\ 0,\ 0,\ 1$	0	H
$a1$	$1\times(4,1,1,\overline{2},1,1)$	1	0	0	$\frac{1}{6},\ -\frac{1}{2}$	$\frac{1}{3},\ -1$	
$a2$	$1\times(\overline{4},1,1,1,2,1)$	-1	0	0	$-\frac{1}{6},\ \frac{1}{2}$	$-\frac{1}{3},\ 1$	
$b2$	$1\times(1,2,1,1,\overline{2},1)$	0	1	0	$\pm\frac{1}{2}$	0	
$c1$	$2\times(1,1,\overline{2},2,1,1)$	0	0	-1	$\pm\frac{1}{2}$	0	
$c3$	$3\times(1,1,2,1,1,\overline{2})$	0	0	1	$\pm\frac{1}{2}$	0	
$b_{\Box\Box}$	$2\times(1,3,1,1,1,1)$	0	2	0	$0,\pm1$	0	
$b_{\genfrac{}{}{0pt}{}{\Box}{\Box}}$	$2\times(1,\overline{1},1,1,1,1)$	0	-2	0	0	0	
$c_{\overline{\Box\Box}}$	$1\times(1,1,\overline{3},1,1,1)$	0	0	-2	$0,\pm1$	0	
$c_{\genfrac{}{}{0pt}{}{\Box}{\Box}}$	$1\times(1,1,1,1,1,1)$	0	0	2	0	0	

supersymmetry. For the models with two confining $USp(N)$ gauge groups, a general analysis of the nonperturbative superpotential with tree-level gauge couplings shows [33] that there can be extrema with the dilaton and complex structure moduli stabilized, however, the extrema are saddle points in general and they do not break supersymmetry in general. On the other hand, for the models with three or four confining $USp(N)$ gauge groups, the non-perturbative superpotientil in general allows for the stabilization of moduli and breaking of supersymmetry at the stable extremum. [For an explicit analysis of three confining USp gauge group factors see Ref. [33].] In our case, six models (I-NZ-1a, I-Z-2, I-Z-3, I-Z-7, I-Z-8 and I-Z-9) have two $USp(N)$ gauge groups with negative beta functions, three models (I-Z-1, I-Z-4 and I-Z-5) have three confining $USp(N)$ gauge groups, and two models (I-Z-6 and I-Z-10) have four confining $USp(N)$ gauge groups. Therefore, for the latter five models, due to gaugino condensations there may be stable extrema with the stabilized moduli and broken supersymmetry. These five models are also very interesting from other phenomenological points of view. Note also that similar to the case studied in Ref. [33], at the extremum the cosmological constant is likely to be negative and close to the string scale, and thus in these models the gaugino condensation is not likely to address the cosmological constant problem.

We should emphasize that all the models possess exotic particles charged under the hidden gauge group factors. There is a possibility that the strong coupling dynamics of the hidden sector at some intermediate scale would provide a mechanism for all of these particles

TABLE V: The chiral spectrum in the open string sector of model I-Z-10

I-Z-10	$SU(4)\times SU(2)_L\times SU(2)_R\times USp(2)^4$	Q_4	Q_{2L}	Q_{2R}	Q_{em}	$B-L$	Field
ab	$3\times(4,\overline{2},1,1,1,1,1)$	1	-1	0	$-\frac{1}{3},\ \frac{2}{3},\ -1,\ 0$	$\frac{1}{3},\ -1$	Q_L, L_L
ac	$3\times(\overline{4},1,2,1,1,1,1)$	-1	0	1	$\frac{1}{3},\ -\frac{2}{3},\ 1,\ 0$	$-\frac{1}{3},\ 1$	Q_R, L_R
$a1$	$1\times(4,1,1,2,1,1,1)$	1	0	0	$\frac{1}{6},\ -\frac{1}{2}$	$\frac{1}{3},\ -1$	
$a2$	$1\times(\overline{4},1,1,1,2,1,1)$	-1	0	0	$-\frac{1}{6},\ \frac{1}{2}$	$-\frac{1}{3},\ 1$	
$b2$	$1\times(1,2,1,1,2,1,1)$	0	1	0	$\pm\frac{1}{2}$	0	
$b4$	$3\times(1,\overline{2},1,1,1,1,2)$	0	-1	0	$\mp\frac{1}{2}$	0	
$c1$	$1\times(1,1,\overline{2},2,1,1,1)$	0	0	-1	$\pm\frac{1}{2}$	0	
$c3$	$3\times(1,1,2,1,1,2,1)$	0	0	1	$\pm\frac{1}{2}$	0	
$b_{\square\square}$	$2\times(1,3,1,1,1,1,1)$	0	2	0	$0,\pm1$	0	
$b_{\overline{\substack{\square\\ \square}}}$	$2\times(1,\overline{1},1,1,1,1,1)$	0	-2	0	0	0	
$c_{\overline{\square\square}}$	$2\times(1,1,\overline{3},1,1,1,1)$	0	0	-2	$0,\pm1$	0	
$c_{\substack{\square\\ \square}}$	$2\times(1,1,1,1,1,1,1)$	0	0	2	0	0	

to form bound states or composite particles (compatible with anomaly cancellation constraints); these particles would in turn be charged only under the SM gauge symmetry [30], which is similar to the quark condensation in QCD. Generally speaking, USp groups have two kinds of neutral bound states. The first one is the pseudo inner product of two fundamental representations, which is generated by reducing the rank 2 anti-symmetric representation

TABLE VI: The composite particle spectrum of Model I-Z-10, which is formed due to the strong forces from hidden sector.

Model I-Z-10		$SU(4)\times SU(2)_L\times SU(2)_R\times USp(2)^4$	
Confining Force	Intersection	Exotic Particle Spectrum	Confined Particle Spectrum
$USp(2)_1$	$a1$	$1\times(4,1,1,2,1,1,1)$	$1\times(4^2,1,1,1,1,1,1)$, $1\times(4,1,\overline{2},1,1,1,1)$
	$c1$	$1\times(1,1,\overline{2},2,1,1,1)$	$1\times(1,1,\overline{2}^2,1,1,1,1)$
$USp(2)_2$	$a2$	$1\times(\overline{4},1,1,1,2,1,1)$	$1\times(\overline{4}^2,1,1,1,1,1,1)$, $1\times(\overline{4},2,1,1,1,1,1)$
	$b2$	$1\times(1,2,1,1,2,1,1)$	$1\times(1,2^2,1,1,1,1,1)$
$USp(2)_3$	$c3$	$3\times(1,1,2,1,1,2,1)$	$6\times(1,1,2^2,1,1,1,1)$
$USp(2)_4$	$b4$	$3\times(1,\overline{2},1,1,1,1,2)$	$6\times(1,\overline{2}^2,1,1,1,1,1)$

and is generic for USp groups. This is somehow the reminiscent of a meson, which is the inner product of one pair of $SU(3)$ fundamental and anti-fundamental representations in QCD. The second one is rank $2N$ anti-symmetric representation for $USp(2N)$ group with $N \geq 2$, which is also a singlet under $USp(2N)$ transformation and is similar to a baryon, a rank 3 anti-symmetric representation in QCD. A definite example containing such a singlet is model I-Z-2 whose groups in the hidden sector are $USp(4) \times USp(4)$. For $N = 1$, this singlet is identified with the first one. In order to explicitly show how to form the composite states, one concrete example from Model I-Z-10 is given out, with the confined particle spectra tabulated in Table VI. Model I-Z-10 has four confining gauge groups. Thereinto, both $USp(2)_1$ and $USp(2)_2$ have two charged intersections. So for them, besides self-confinement, the mixed-confinement between different intersections is also possible, which yields the chiral supermultiplets $(4, 1, \overline{2}, 1, 1, 1, 1)$ and $(\overline{4}, 2, 1, 1, 1, 1, 1)$. As for $USp(2)_3$ and $USp(2)_4$, they have only one charged intersection. Thus, there is no mixed-confinement, and self-confinement leads to 6 tensor representations for both ones. In addition, it is not hard to check from the spectra that no new anomaly is introduced to the remaining gauge symmetry, *i.e.*, this model is still anomaly-free. This set of analysis works as well for the other models except Model I-NZ-1(a-c) and Model I-Z-8 where a non-asymptotical free gauge symmetry appears in the hidden sector, and the states charged under this symmetry have no chance to be confined. By the way, the anomaly-cancellation for the confined particle spectra is not automatically guaranteed. Sometimes one additional field associated with composite states is also required to satisfy t' Hooft anomaly matching condition. Here, we only consider one relatively simple example in order to avoid the unnecessary complications.

VI. SOME OTHER POTENTIALLY INTERESTING SETUPS

In addition to the models that we have discussed, there are some other potentially interesting constructions that could lead to the SM constructions. For example, if we assume

$$I_{ac} = -(3 + h) \ , \ I_{ac'} = h \ , \tag{182}$$

with h a positive integer, we can have massless vector-like Higgs fields which can break the Pati-Salam gauge symmetry down to the SM gauge symmetry or break the $U(1)_{B-L} \times U(1)_{I_{3R}}$ down to $U(1)_Y$. But, due to the large wrapping numbers required by the increased ABSes of I_{ac} and $I_{ac'}$, it is very difficult to find such models. For instance, let us focus on the $h = 1$ case with one tilted two-torus. Considering that all factors of $2^k I_{ac} = -8$ and $2^k I_{ac'} = 2$ are even, they should come from the 3rd tilted two-torus, *i.e.*, we should have $|n_a^3 l_c^3| = 5$ or 3 and $|n_c^3 l_a^3| = 3$ or 5. This implies that the 1st and 2nd WNPs will contribute a unit factor to both $2^k I_{ac} = -8$ and $2^k I_{ac'} = 2$, and thus we must have $n_a^1 l_a^1 = 0$ and $n_c^2 l_c^2 = 0$. Therefore

both a- and c-branes are of Z-type. If $|n_a^3 l_a^3| = 15$, obviously there is no solution due to the TCC violation. If $|n_c^3 l_c^3| = 15$, given $|n_b^3 l_b^3| = 1$ to generate the even factors of $2^k I_{ab} = 6$ and $2^k I_{ab'} = 0$ and the constraints from BTCC and CTCC, the ATCC cannot be satisfied. If $|n_a^3 l_a^3| = 5$, to avoid the TCC violation, the even factors of $2^k I_{ab} = 6$ and $2^k I_{ab'} = 0$ can not be generated from the third WNPs at the same time. As for $|n_a^3 l_a^3| = 3$, there are less than two asymptotically free USp groups available in order to satisfy ATCC or BTCC for $|n_a^3 n_c^3| = 15$ or $|l_a^3 l_c^3| = 15$, respectively. In short, it is hard to find such models.

Another interesting possibility would correspond to constructions where the $SU(2)_L$ and/or $SU(2)_R$ gauge symmetries come from filler branes, *i.e.* the $SU(2)_{L,R} = USp(2)_{L,R}$. In this case the number of the SM Higgs doublet pairs may be decreased. However, we do not want to obtain $SU(2)_{L,R}$ from the splittings of higher rank $USp(N)$ ($N \geq 4$) branes which would generically lead to even number of families. In this case, one wrapping number with its ABS larger than 2 cannot be avoided for the $U(4)$ brane. This may make the model building very hard due to tadpole cancellation constraints. However, further exploration of these models is needed.

VII. DISCUSSIONS AND CONCLUSIONS

In this paper we reviewed the rules for supersymmetric model building, and the conditions for the tadpole cancellations and 4-dimensional $N = 1$ supersymmetric D6-brane configurations in the Type IIA theory on $T^6/(\mathbf{Z_2} \times \mathbf{Z_2})$ orientifold with D6-brane intersections. Subsequently, we highlighted the interesting features of the three-family supersymmetric $SU(4)_C \times SU(2)_L \times SU(2)_R$ models where all the gauge symmetries arise from the stacks of D6-branes with $U(n)$ gauge symmetries. In particular, we demonstrated that the Pati-Salam gauge symmetry can be broken down to the $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ via D6-brane splittings, and further down to the Standard Model gauge symmetry via the D- and F-flatness preserving Higgs mechanism where Higgs fields arise from the massless open string states in a specific $N = 2$ subsector of the theory. In order to stabilize the (complex structure) modulus and provide a possibility to break supersymmetry via a "race-track" scenario, we required that there be at least two confining USp groups in the hidden sector.

In order to facilitate the systematic search, we discussed the T-duality and its variations that are in effect for the intersecting D6-brane constructions on Type IIA orientifolds. Employing these symmetries allowed us to search for all inequivalent models with above specified properties. We found no models in the case when zero, two, or three two-tori are tilted. For the case with only one tilted two-torus, we obtain 11 inequivalent models. Eight of them have 8 or fewer pairs of the SM Higgs doublets. Especially, one model has only 2 pairs of the SM Higgs doublets. Furthermore, two models have the gauge coupling

unification for $SU(2)_L$ and $SU(2)_R$ gauge group factors at the string scale and the Higgs pairs for them arise from the massless open string states in a $N = 2$ subsector. The explicit brane configurations, their intersections, the gauge group structure (and the hidden sector beta functions) are tabulated for all these models in the Appendix. As explicit examples, we also present the chiral spectra in the open string sector for the models I-NZ-1a, I-Z-6 and I-Z-10. We also briefly comment on the other potentially interesting configurations of intersecting D6-branes which could lead to the three family supersymmetric Standard Model. In particular, the setup when the origin of $SU(2)_L$ and/or $SU(2)_R$ comes from the USp brane configurations is extremely constraining; it seems to be very difficult to find the supersymmetric three-family models of this type.

Models presented in this paper provide a promising stepping stone toward the realistic SM models from string theory. In particular, the symmetry breaking chain of the original Pati-Salam models (via brane splitting and subsequent Higgs mechanism) allows for obtaining only the Standard Model gauge group structure at electroweak scale. While we made some preliminary comments on the phenomenological features of these models constructed, there are a number of avenues opened for further study.

One should study the renormalization group equations for the running of the gauge couplings, both in the observable and hidden sectors. We expect that due to the exotic matters and the additional adjoint chiral superfields, the low energy predictions for the SM gauge couplings may not be consistent with these from experiments. Note however, that the left-right gauge coupling unification at the string scale for some models may provide interesting consequences for the low energy couplings.

The issue of supersymmetry breaking and modulus stabilization via gaugino condensation in the hidden sector (with at least two confining USp gauge group factors) should be addressed in detail. If supersymmetry breaking does take place in these models, this breaking can be mediated via gauge interactions, thus providing a possibility to address the CP problem in this framework. In addition, the nature of the soft supersymmetry breaking parameters and their model dependence, deserve further detailed study. Another important role of the strong dynamics in the hidden sector could play, is to bind fractionally charged exotic matter into the composite objects with only SM quantum numbers [30].

An important topic is further study of the Higgs mechanism (that preserves D- and F-flatness condition) for the breaking of $U(1)_{B-L} \times U(1)_{I_{3R}}$ down to $U(1)_Y$. In order to be consistent with the see-saw mechanism to explain the tiny neutrino masses, and with the leptogenesis to produce the baryon asymmetry, we need the symmetry breaking scale at about 10^{15} GeV which hopefully can be realized in our models (with radiative corrections fixing this scale as one possibility).

The study of the SM fermion masses and mixings is also important. Yukawa couplings

can be calculated exactly in conformal field theory [32] and they have a beautiful geometric interpretation in terms of the angles and areas of the triangles, specified by the location of brane intersections in the internal space. In particular, the models with few SM Higgs pairs should provide an important framework to address the textures of the SM fermion mass matrices in detail [44].

Acknowledgments

We would like to thank Paul Langacker for useful discussions. The research was supported in part by the National Science Foundation under Grant No. INT02-03585 (MC) and PHY-0070928 (T. Li), DOE-EY-76-02-3071 (MC, T. Liu) and Fay R. and Eugene L. Langberg Chair (MC).

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Appendix: Tables for Supersymmetric Pati-Salam Models

In Appendix we tabulate all 11 inequivalent models, found by the systematic search. Thereinto, we present only its equivalence class, specified by T-dualities. In the first column of each table, a, b and c denote the $U(4)$, $U(2)_L$, and $U(2)_R$ stacks of branes, respectively. 1, 2, 3, and 4 represent the filler branes along respective ΩR, $\Omega R\omega$, $\Omega R\theta\omega$ and $\Omega R\theta$ orientifold planes, resulting in $USp(N)$ gauge groups. N in the second column is the number of D6-branes in each stack. The third column shows the wrapping numbers of the various branes, and we have specified the third set of wrapping number for the tilted two-torus. (Recall, only one two-torus is tilted.) The intersection numbers between the various stacks are given in the remaining right columns where b' and c' are respectively the ΩR images of b and c. For convenience, we also tabulate the relation among the moduli parameters imposed by the supersymmetry conditions, as well as the β functions (β_i^g) of the gauge groups in the hidden sector. Again, we required at least two asymptotically free USp groups with negative β functions in the hidden sector. For example, model I-NZ-1a, which forms one equivalent class together with I-NZ-1b and I-NZ-1c, has two asymptotically free USp groups among its three USp groups in the hidden sector. Moreover, I-, -Z- and -NZ- imply that only one two-torus is tilted, a-brane is Z-type, and a-brane is NZ-type, respectively.

TABLE VII: D6-brane configurations and intersection numbers for the three-family left-right symmetric model I-NZ-1a

model I-NZ-1a	$U(4)\times U(2)_L\times U(2)_R\times USp(2)\times USp(4)\times USp(2)$										
stack	N	$(n^1,l^1)\times(n^2,l^2)\times(n^3,l^3)$	$n_{\square\square}$	$n_{\begin{smallmatrix}\square\\ \square\end{smallmatrix}}$	b	b'	c	c'	1	2	3
a	8	$(1,-1)\times(1,1)\times(1,1)$	0	4	3	0	-3	0	1	-1	1
b	4	$(0,1)\times(1,-2)\times(1,-1)$	1	-1	-	-	0	2	-2	1	0
c	4	$(1,0)\times(1,4)\times(1,-1)$	-3	3	-	-	-	-	0	0	4
1	2	$(1,0)\times(1,0)\times(2,0)$	$X_A=\frac{1}{2}X_B=5X_C=\frac{5}{4}X_D$								
2	4	$(1,0)\times(0,-1)\times(0,2)$	$(\chi_1=5/\sqrt{2},\chi_2=1/2\sqrt{2},\chi_3=2\sqrt{2})$								
3	2	$(0,-1)\times(1,0)\times(2,0)$	$\beta_1^g=-2,\beta_2^g=-3,\beta_3^g=0$								

TABLE VIII: D6-brane configurations and intersection numbers for the three-family left-right symmetric model I-NZ-1b

model I-NZ-1b	$U(4) \times U(2)_L \times U(4)_R \times USp(2) \times USp(2) \times USp(2)$											
stack	N	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$	$n_{\Box\Box}$	$n_{\substack{\Box\\\Box}}$	b	b'	c	c'	2	3	4	
a	8	$(1, -1) \times (1, 1) \times (1, 1)$	0	4	3	0	-3	0	-1	1	1	
b	4	$(2, 1) \times (1, 0) \times (1, -1)$	1	-1	-	-	0	2	1	0	-2	
c	4	$(4, -1) \times (0, 1) \times (1, -1)$	-3	3	-	-	-	-	0	4	0	
2	4	$(1, 0) \times (0, -1) \times (0, 2)$	$X_A = \frac{2}{5} X_B = 4X_C = \frac{4}{5} X_D$									
3	2	$(0, -1) \times (1, 0) \times (2, 0)$	$(\chi_1 = 2\sqrt{2}, \chi_2 = \sqrt{2}/5, \chi_3 = 2\sqrt{2})$									
4	2	$(0, -1) \times (0, 1) \times (2, 0)$	$\beta^g_2 = -3, \beta^g_3 = 0, \beta^g_4 = -2$									

TABLE IX: D6-brane configurations and intersection numbers for the three-family left-right symmetric model I-NZ-1c

model I-NZ-1c	$U(4) \times U(2)_L \times U(2)_R \times USp(4) \times USp(2) \times USp(2)$											
stack	N	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$	$n_{\Box\Box}$	$n_{\substack{\Box\\\Box}}$	b	b'	c	c'	1	2	4	
a	8	$(-1, -1) \times (1, 1) \times (1, 1)$	0	-4	3	0	-3	0	1	-1	-1	
b	4	$(1, 0) \times (4, 1) \times (1, -1)$	3	-3	-	-	0	-2	0	0	-4	
c	4	$(0, 1) \times (2, -1) \times (1, -1)$	-1	1	-	-	-	-	-1	2	0	
1	4	$(1, 0) \times (1, 0) \times (2, 0)$	$X_A = 2X_B = \frac{5}{2} X_C = 10X_D$									
2	2	$(1, 0) \times (0, -1) \times (0, 2)$	$(\chi_1 = 5/\sqrt{2}, \chi_2 = 2\sqrt{2}, \chi_3 = \sqrt{2})$									
4	2	$(0, -1) \times (0, 1) \times (2, 0)$	$\beta^g_1 = -3, \beta^g_2 = -2, \beta^g_4 = 0$									

TABLE X: D6-brane configurations and intersection numbers for the three-family left-right symmetric model I-Z-1

model I-Z-1	$U(4) \times U(2)_L \times U(2)_R \times USp(2) \times USp(2) \times USp(2)$										
stack	N	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$	$n_{\Box\Box}$	$n_{\substack{\Box \\ \Box}}$	b	b'	c	c'	1	2	3
a	8	$(0, -1) \times (1, 1) \times (1, 1)$	0	0	3	0	-3	0	1	-1	0
b	4	$(1, 0) \times (4, 1) \times (1, -1)$	3	-3	-	-	0	-7	0	0	1
c	4	$(-1, 1) \times (1, -2) \times (1, -1)$	2	-6	-	-	-	-	-2	1	2
1	2	$(1, 0) \times (1, 0) \times (2, 0)$	$X_A = X_B = \frac{9}{4} X_C = 9 X_D$								
2	2	$(1, 0) \times (0, -1) \times (0, 2)$	$\beta^g_1 = -2, \beta^g_2 = -3, \beta^g_3 = -3$								
3	2	$(0, -1) \times (1, 0) \times (2, 0)$									

TABLE XI: D6-brane configurations and intersection numbers for the three-family left-right symmetric model I-Z-2

model I-Z-2	$U(4) \times U(2)_L \times U(2)_R \times USp(4) \times USp(4)$									
stack	N	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$	$n_{\Box\Box}$	$n_{\substack{\Box \\ \Box}}$	b	b'	c	c'	1	2
a	8	$(0, -1) \times (1, 1) \times (1, 1)$	0	0	3	0	-3	0	1	-1
b	4	$(3, 1) \times (1, 0) \times (1, -1)$	2	-2	-	-	0	-4	0	1
c	4	$(-1, 1) \times (1, -2) \times (1, -1)$	2	6	-	-	-	-	-2	1
1	4	$(1, 0) \times (1, 0) \times (2, 0)$	$X_A = X_B = 3 X_C = 3 X_D$							
2	4	$(1, 0) \times (0, -1) \times (0, 2)$	$\beta^g_1 = -2, \beta^g_2 = -2$							

TABLE XII: D6-brane configurations and intersection numbers for the three-family left-right symmetric model I-Z-3

model I-Z-3	$U(4) \times U(2)_L \times U(2)_R \times USp(2) \times USp(4)$									
stack	N	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$	$n_{\Box\Box}$	$n_{\substack{\Box \\ \Box}}$	b	b'	c	c'	2	4
a	8	$(0, -1) \times (1, 1) \times (1, 1)$	0	0	3	0	-3	0	0	0
b	4	$(3, 1) \times (1, 0) \times (1, -1)$	2	-2	-	-	0	4	0	-3
c	4	$(1, 0) \times (1, 4) \times (1, -1)$	-3	3	-	-	-	-	4	-1
3	2	$(0, -1) \times (1, 0) \times (0, 2)$	$X_A = X_B = 12 X_C = 3 X_D$							
4	4	$(0, -1) \times (0, 1) \times (2, 0)$	$\beta^g_3 = -2, \beta^g_4 = -2$							

TABLE XIII: D6-brane configurations and intersection numbers for the three-family left-right symmetric model I-Z-4

model I-Z-4	$U(4)\times U(2)_L\times U(2)_R\times USp(2)\times USp(2)\times USp(2)$											
stack	N	$(n^1,l^1)\times(n^2,l^2)\times(n^3,l^3)$	$n_{\square\square}$	$n_{\substack{\square\\ \square}}$	b	b'	c	c'	1	2	3	
a	8	$(0,-1)\times(1,1)\times(1,1)$	0	0	2	1	-3	0	1	-1	0	
b	4	$(1,1)\times(1,0)\times(3,-1)$	-2	2	-	-	5	2	0	3	0	
c	4	$(3,-2)\times(0,1)\times(1,-1)$	-1	1	-	-	-	-	-2	0	3	
1	2	$(1,0)\times(1,0)\times(2,0)$	$X_A=X_B=\frac{3}{2}X_C=\frac{1}{3}X_D$									
2	2	$(1,0)\times(0,-1)\times(0,2)$	$\beta^g_1=-2,\beta^g_2=-1,\beta^g_3=-3$									
3	2	$(0,-1)\times(1,0)\times(2,0)$										

TABLE XIV: D6-brane configurations and intersection numbers for the three-family left-right symmetric model I-Z-5

model I-Z-5	$U(4)\times U(2)_L\times U(2)_R\times USp(2)\times USp(2)\times USp(2)$											
stack	N	$(n^1,l^1)\times(n^2,l^2)\times(n^3,l^3)$	$n_{\square\square}$	$n_{\substack{\square\\ \square}}$	b	b'	c	c'	1	2	3	
a	8	$(0,-1)\times(1,1)\times(1,1)$	0	0	3	0	-3	0	1	-1	0	
b	4	$(3,1)\times(1,0)\times(1,-1)$	2	-2	-	-	0	-3	0	1	0	
c	4	$(3,-2)\times(0,1)\times(1,-1)$	-1	1	-	-	-	-	-2	0	3	
1	2	$(1,0)\times(1,0)\times(2,0)$	$X_A=X_B=\frac{3}{2}X_C=3X_D$									
2	2	$(1,0)\times(0,-1)\times(0,2)$	$\beta^g_1=-2,\beta^g_2=-3,\beta^g_3=-3$									
3	2	$(0,-1)\times(1,0)\times(2,0)$										

TABLE XV: D6-brane configurations and intersection numbers for the three-family left-right symmetric model I-Z-6

model I-Z-6	$U(4) \times U(2)_L \times U(2)_R \times USp(2) \times USp(2) \times USp(2) \times USp(2)$											
stack	N	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$	$n_{\square\square}$	$n_{\substack{\square \\ \square}}$	b	b'	c	c'	1	2	3	4
a	8	$(0, -1) \times (1, 1) \times (1, 1)$	0	0	2	1	-3	0	1	-1	0	0
b	4	$(1, 1) \times (1, 0) \times (3, -1)$	-2	2	-	-	4	4	0	3	0	-1
c	4	$(3, -1) \times (0, 1) \times (1, -1)$	-2	2	-	-	-	-	-1	0	3	0
1	2	$(1, 0) \times (1, 0) \times (2, 0)$	$X_A = X_B = 3X_C = \frac{1}{3} X_D$									
2	2	$(1, 0) \times (0, -1) \times (0, 2)$	$\beta^g_1 = -3, \beta^g_2 = -1, \beta^g_3 = -3, \beta^g_4 = -5$									
3	2	$(0, -1) \times (1, 0) \times (0, 2)$										
4	2	$(0, -1) \times (0, 1) \times (2, 0)$										

TABLE XVI: D6-brane configurations and intersection numbers for the three-family left-right symmetric model I-Z-7

model I-Z-7	$U(4) \times U(2)_L \times U(2)_R \times USp(2) \times USp(4)$									
stack	N	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$	$n_{\square\square}$	$n_{\substack{\square \\ \square}}$	b	b'	c	c'	3	4
a	8	$(0, -1) \times (1, 1) \times (1, 1)$	0	0	2	1	-3	0	0	0
b	4	$(1, 1) \times (1, 0) \times (3, -1)$	-2	2	-	-	4	8	0	-1
c	4	$(1, 0) \times (1, 4) \times (1, -1)$	-3	3	-	-	-	-	4	-1
3	2	$(0, -1) \times (1, 0) \times (0, 2)$	$X_A = X_B = \frac{4}{3} X_C = \frac{1}{3} X_D$							
4	4	$(0, -1) \times (0, 1) \times (2, 0)$	$\beta^g_3 = -2, \beta^g_4 = -4$							

TABLE XVII: D6-brane configurations and intersection numbers for the three-family left-right symmetric model I-Z-8

model I-Z-8	$U(4) \times U(2)_L \times U(2)_R \times USp(2) \times USp(2) \times USp(2)$											
stack	N	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$	$n_{\Box\Box}$	$n_{\substack{\Box\\\Box}}$	b	b'	c	c'	1	2	4	
a	8	$(0, -1) \times (1, 1) \times (1, 1)$	0	0	2	1	-3	0	1	-1	0	
b	4	$(1, 2) \times (1, 0) \times (3, -1)$	-5	5	-	-	7	10	0	6	-1	
c	4	$(3, -1) \times (0, 1) \times (1, -1)$	-2	2	-	-	-	-	-1	0	0	
1	2	$(1, 0) \times (1, 0) \times (2, 0)$	$X_A = X_B = 3X_C = \frac{1}{6} X_D$									
2	2	$(1, 0) \times (0, -1) \times (0, 2)$	$\beta_1^g = -3, \beta_2^g = 2, \beta_4^g = -5$									
4	2	$(0, -1) \times (0, 1) \times (2, 0)$										

TABLE XVIII: D6-brane configurations and intersection numbers for the three-family left-right symmetric model I-Z-9

model I-Z-9	$U(4) \times U(2)_L \times U(2)_R \times USp(2) \times USp(2)$										
stack	N	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$	$n_{\Box\Box}$	$n_{\substack{\Box\\\Box}}$	b	b'	c	c'	1	2	
a	8	$(0, -1) \times (1, 1) \times (1, 1)$	0	0	3	0	-3	0	1	-1	
b	4	$(3, 2) \times (1, 0) \times (1, -1)$	-1	1	-	-	0	0	0	2	
c	4	$(3, -2) \times (0, 1) \times (1, -1)$	1	-1	-	-	-	-	-2	0	
1	2	$(1, 0) \times (1, 0) \times (2, 0)$	$X_A = X_B = \frac{3}{2} X_C = \frac{3}{2} X_D$								
2	2	$(1, 0) \times (0, -1) \times (0, 2)$	$\beta_1^g = -2, \beta_2^g = -2,$								

TABLE XIX: D6-brane configurations and intersection numbers for the three-family left-right symmetric model I-Z-10

model I-Z-10	$U(4) \times U(2)_L \times U(2)_R \times USp(2) \times USp(2) \times USp(2) \times USp(2)$											
stack	N	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$	$n_{\square\square}$	$n_{\substack{\square \\ \square}}$	b	b'	c	c'	1	2	3	4
a	8	$(0, -1) \times (1, 1) \times (1, 1)$	0	0	3	0	-3	0	1	-1	0	0
b	4	$(3, 1) \times (1, 0) \times (1, -1)$	2	-2	-	-	0	0	0	1	0	-3
c	4	$(3, -1) \times (0, 1) \times (1, -1)$	-2	2	-	-	-	-	-1	0	3	0
1	2	$(1, 0) \times (1, 0) \times (2, 0)$	$X_A = X_B = 3X_C = 3X_D$									
2	2	$(1, 0) \times (0, -1) \times (0, 2)$	$\beta_1^g = -3, \beta_2^g = -3, \beta_3^g = -3, \beta_4^g = -3$									
3	2	$(0, -1) \times (1, 0) \times (0, 2)$										
4	2	$(0, -1) \times (0, 1) \times (2, 0)$										